International Journal of Heat and Mass Transfer 125 (2018) 815–827

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

The natural convection in a square enclosure with two hot inner cylinders, Part I: The effect of one elliptical cylinder with various aspect ratios in a vertical array

Hyun Woo Cho [a], Yong Gap Park [b], Man Yeong Ha [a,]*

[a] School of Mechanical Engineering, Pusan National University, Jang Jeon 2-Dong, Geum Jeong Gu, Busan 609-735, Republic of Korea
[b] Rolls-Royce and Pusan National University Technology Centre in Thermal Management, Jang Jeon 2-Dong, Geum Jeong Gu, Busan 609-735, Republic of Korea

ARTICLE INFO

Article history:
Received 12 February 2018
Received in revised form 24 April 2018
Accepted 26 April 2018

Keywords:
Natural convection
Immersed boundary method
Vertical array
Elliptical cylinder

ABSTRACT

This study numerically investigates the two-dimensional natural convection in a square enclosure with a vertical array of one circular cylinder and one elliptical cylinder at Rayleigh numbers of $10^4 \le Ra \le 10^6$. A simulation was carried out based on the immersed boundary method to obtain an accurate solution. The effects of the aspect ratio of the elliptical cylinder in the range of $0.25 \le AR \le 4.00$ were investigated. When the Rayleigh number increases to $Ra = 10^6$, the numerical solutions reach an unsteady state for all cases of the lower elliptical cylinder and the case of the upper elliptical cylinder except at $AR = 2.00$ and $AR = 4.00$. The transition of the flow regime from the unsteady state to the steady state depends on the variation in the ratio of the elliptical cylinder. In addition, in the case of the upper elliptical cylinder at $AR = 0.50$ the time- and surface-averaged Nusselt numbers for the walls of the enclosure increase by 2.1% in the case of two circular cylinders at $Ra = 10^6$.

1. Introduction

Natural convection in an enclosure has been a focused on the numerous applications, such as nuclear and chemical reactors, cooling of electronic equipment, and heat exchangers. Buoyancy-driven heat transfer exhibits a great variety of complex dynamic behaviors, which depend on the temperature difference between the inner bodies and outer bodies, the shapes and sizes of the inner bodies, and their position and arrangement [1–23]. There has been relatively little investigation on the natural convection between an enclosure and a heated elliptical cylinder. Bararnia et al. [24] studied the natural convection around a horizontal elliptical cylinder inside a square enclosure using the lattice Boltzmann method (LBM). They investigated the hydrodynamic and thermal behaviors of the fluid at various vertical positions of the inner elliptical cylinder for Rayleigh numbers of 10^3–10^6. In addition, the average Nusselt number increased with the Rayleigh number and the minimum value occurred at a greater distance the top of the inner cylinder with increasing Rayleigh number. Zhang et al. [25] investigated the influence of the inclination angle of the outer square enclosure and the size of the inner cylinder. The inclination angle of the outer enclosure and the size of the inner cylinder have remarkable effects on the flow and thermal structures in the enclosure. However, the inclination angle of the enclosure had a small effect on the average Nusselt number on the inner elliptical cylinder.

Liao and Lin [26] studied the natural and mixed convection using an immersed-boundary method. They investigated the influence of the Rayleigh number, axis ratio, and inclination angle of the elliptical cross section. The local and averaged heat transfer characteristics around the surfaces of both the inner cylinder and outer enclosure were investigated, and the various parameters had remarkable effects on the heat transfer performance. Park et al. [27] looked at the influence of different vertical locations of two inner cylinders. The cylinders were equally moved in a vertical array, and Rayleigh numbers of $10^3 \le Ra \le 10^6$ were investigated. This study concentrated on the effects of the gap between the cylinders on the natural convection characteristics. The flow regime for unsteady state occurred at $Ra = 10^6$ and when the distance between the cylinders was $\delta_v = 0.3L$. The regime was strongly affected by the Rayleigh number and the distance between the cylinders.

In recent years, the flow and heat transfer characteristics are heavily affected by the multiple circular cylinder and the location [20–23]. In addition, many researchers have investigated the effect of the aspect ratio of one elliptical cylinder, inclination angle, and variations in arrays of two inner circular cylinders. However, it is

* Corresponding author.
E-mail address: myha@pusan.ac.kr (M.Y. Ha).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.04.141
0017-9310/

Nomenclature

Symbols

AR	aspect ratio
f_i	momentum forcing
g	acceleration of gravity
L	length of square enclosure
n	normal direction to the wall
Nu	local Nusselt number
$\overline{Nu}$	time-averaged Nusselt number
$\langle Nu \rangle$	surface-averaged Nusselt number
$\overline{\langle Nu \rangle}$	time- and surface-averaged Nusselt number
P	dimensionless pressure
Pr	Prandtl number
q	mass source and sink
R	radius of circular cylinder
Ra	Rayleigh number
t	dimensionless time
u_i	dimensionless velocity
W	surface area of walls
x_i	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
δ_{i2}	kronecker delta
ρ	density
ν	kinematic viscosity
θ	dimensionless temperature

Superscripts/subscripts

$*$	dimensional value
B	bottom wall
c	cold
C	cylinder
En	enclosure
h	hot
L	left wall
R	right wall
T	top wall

hard to find numerical research on the natural convection from one elliptical cylinder and one circular cylinder in a vertical array. Thus, this study focuses on the effect of the positions of elliptical cylinder and aspect ratio in a vertical array for Rayleigh numbers of $10^4 \leq Ra \leq 10^6$ and a Prandtl number of $Pr = 0.7$. The transition of the flow regime from the unsteady state to the steady state was analyzed for different ratios of the elliptical cylinder. In addition, the flow and thermal fields and the time- and surface-averaged Nusselt numbers were analyzed according to the location of one inner elliptical cylinder and the aspect ratio.

2. Computation details

2.1. Numerical methods

From the industrial application perspective, the geometry is complex and the grid generating is one of the difficulty in the computational fluid mechanics. Thus, some numerical methods have been developed to implement the numerical analysis on the phenomena occurred in the complex geometries. The most popular method among them is the IBM and it has been used by many researchers to investigate the numerous cases with complex geometries [28-30]. The immersed boundary method is easier to implement and more efficient than classical approaches such as body-fitted curvilinear grids. Especially, the IBM has the advantage at analyzing the fluid flow pattern in the system with numerous inner objects or moving objects.

An elliptical cylinder is a basic and general form that can become a flat plate or a circular cylinder depending on the aspect ratio (AR). Even though its shape changes for various AR, the same simple Cartesian coordinate system can be used for simulating the flow over the cylinder. Thus, this method was used to handle the surface of the cylinders in the square enclosure. Kim et al. [31], Kim and Choi [32], and Choi [33] describe further details about the immersed boundary method.

The governing equations are the continuity, momentum, and energy equations in their non-dimensional forms:

$$\frac{\partial u_i}{\partial x_i} - q = 0 \tag{1}$$

$$\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} = -\frac{\partial P}{\partial x_j} + \sqrt{\frac{Pr}{Ra}} \frac{\partial^2 u_j}{\partial x_i^2} + \theta \delta_{j2} + f_j \tag{2}$$

$$\frac{\partial \theta}{\partial t} + u_i \frac{\partial \theta}{\partial x_i} = \frac{1}{\sqrt{RaPr}} \frac{\partial^2 \theta}{\partial x_i^2} + h \tag{3}$$

The dimensionless variables shown above are defined as follows:

$$\begin{aligned} t &= \sqrt{\frac{g\beta(T_h - T_c)}{L}} t^*, \quad x_i = \frac{x_i^*}{L}, u_i = \frac{u_i^*}{U_{ref}} = \frac{u_i^*}{\sqrt{g\beta L(T_h - T_c)}}, \\ P &= \frac{P^*}{\rho U_{ref}^2} = \frac{P^*}{\rho g \beta L (T_h - T_c)}, \quad \theta = \frac{T - T_c}{T_h - T_c} \end{aligned} \tag{4}$$

In these equations, g, T, β, ρ, and L denote the gravitational acceleration, temperature, thermal expansion coefficient, density, and the characteristic length, respectively. The superscript $*$ in Eq. (4) represents the dimensional variables. x_i is the dimensionless Cartesian coordinate, u_i is the corresponding dimensionless velocity component, t is the dimensionless time, P is the dimensionless pressure, and θ is the dimensionless temperature.

The non-dimensionalization results in two dimensionless parameters: $Pr = \nu/\alpha$ and $Ra = g\beta L^3 (T_h - T_c)/\nu\alpha$, where ν, g, α and β are the kinematic viscosity, gravitational acceleration, thermal diffusivity and thermal expansion coefficient, respectively. The terms q, f_i, and h in Eqs. (1)–(3) are related to the immersed boundary method. The mass source/sink q in Eq. (1) and momentum forcing f_i in Eq. (2) are imposed on the body surface and inside the body to satisfy the no-slip condition and mass conservation in the cell containing the virtual boundary. In Eq. (3), the heat source/sink term h is applied to satisfy the isothermal boundary condition at the virtual boundary. A second-order linear or bilinear interpolation scheme was applied to satisfy the no-slip and isothermal conditions at the immersed boundary.

The spatial discretization of Eqs. (1)–(3) was done using the central difference scheme with second-order accuracy based on the finite volume method. The fractional step method proposed by Choi and Moin [34] was used to simulate the time advancement of the flow field. In the discretization process, the advection terms were treated explicitly using the second-order Adams-Bashforth