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Cascade-like and cyclic heat transfer characteristics affected by enclosure aspect ratios for low Prandtl numbers

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For low Prandtl-number fluids, heat-transfer characteristics under the influence of the aspect ratio for buoyancy-driven recirculating flows in rectangular enclosures (with left hot, right cold, top/bottom insulated walls) behave differently from those for high-*Pr* fluids. At $Ra = 10^6$ and $Pr = 0.025$, as the enclosure widens, time-averaged and hot-wall-spatially-averaged heat transfer first decreases, then cascades downward. Locally, heat transfer peaks at a few locations, and these peaks travel cyclically as time elapses. For completeness, the present study serves as a sequel of a previous high-*Pr* investigation.

1. Introduction

Natural convection in rectangular enclosures has attracted considerable attention due to its engineering and science applications in a wide range, such as geophysics, meteorology, chemical and food processing, solar and nuclear power systems, and Czochralski crystal growth among others. Since the aspect ratio (*AR*) of enclosures can significantly affect both hydrodynamic and thermal characteristics in these phenomena for relatively high-*Pr*'s, much effort has been devoted to studying variations of either the enclosure width [1,2] or the enclosure height [3–24].

Wang et al. [1] have focused on the aspect ratio *AR* in the range of $10^3 \leqslant Ra \leqslant 10^6, 0 < AR \leqslant 1.5$, and $0.707 \leqslant Pr \leqslant 10000$, and have discovered that the average Nusselt number exhibits mildly zigzag behaviors as the enclosure widens. Lamsaadi et al. [2] have studied heat transfer characteristics for constant heat flux in wider horizontal rectangular cavities while stressing non-Newtonian power-law fluids and various enclosure widths. Elder [3] has conducted a comprehensive experimental investigation of a liquid in a rectangular cavities with parameters prescribed in the range of $Ra < 10^8, L \leqslant H \leqslant 60L$ and $Pr = 1000$, in three noticeable regimes. Analytically empirical expressions for different Nusselt numbers associated with tall cavities have been proposed Bejan [4], whose results are consistent well with experimental data [3] and numerical solutions [5]. Both experimental and numerical studies [6–16] for recirculating flows in tall cavities continued to be investigated. Cormack et al. have analytically [17] and numerically [18] investigated free convection in shallow rectangular enclosures. For predicting heat transfer in shallow enclosures, Bejan and Tien [19] have developed comprehensive analytical results for Nusselt numbers corresponding to parallel-flow regime, intermediate-flow regime, and boundary-layer regime, and have demonstrated that their analysis adequately predicted experimental [20] and numerical [17] studies. An experimental investigation with differentially-heated side walls in a water-filled enclosure has been studied by Bejan et al. [21], who have established a limiting condition in which convection starts to play a critical role in the thermal transport. Recently, Turan et al. successively focused on height effects in laminar natural convection of different fluids [22–24], in rectangular cavities with differentially-heated side walls for a range of various values of Rayleigh number for $0.125L \leqslant H \leqslant 8L$.

In the regime of low *Pr*'s, by contrast, most investigations focus on oscillatory instability [25–35]. Busse [25] has investigated the

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Nomenclature

ΔT	temperature difference between hot wall and cold wall ($= T_h - T_c, K$)
Δt	two end times of a time interval that covers approximately ten cycles ($= t_2 - t_1, s$)
$\overline{Nu_{avg}}$	time-averaged and hot-wall-spatially-averaged Nusselt number based on H
$\overline{Q_{cd}}$	time-averaged space-averaged conductive flux along y direction (W/m^2)
$\overline{Q_{cv}}$	time-averaged space-averaged convective flux along y direction (W/m^2)
$\overline{Q_T}$	time-averaged space-averaged total energy flux along y direction (W/m^2)
AR	aspect ratio based on H ($= L/H$)
c_p	specific heat capacity with pressure kept constant (J/(kg K))
g	gravitational acceleration (m/s^2)
H	height of the rectangular enclosure (m)
h	local heat transfer coefficient (W/(m^2 K))
k	thermal conductivity (W/(m K))
L	width of the rectangular enclosure (m)
$Nu(y)$	local Nusselt number on the hot wall ($= hH/k$)
Nu_{avg}	hot-wall-spatially-averaged Nusselt number based on H
p	pressure (Pa)
Pr	Prandtl number ($= \nu/\alpha$)
Ra	Rayleigh number based on H ($= g\beta H^3 \Delta T/(\nu\alpha)$)
T	temperature (K)
t	time (s)
u	x-direction velocity (m/s)
v	y-direction velocity (m/s)
x	x coordinate (m)
y	y coordinate (m)

Greek symbols

α	thermal diffusivity (m^2/s)
β	thermal-expansion coefficient (1/K)
μ	viscosity (kg/(m s))
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)

Subscripts

h	hot
c	cold
i	initial

instability of convection rolls in a fluid layer heated from below for stress-free boundaries in the limit of small Prandtl number. Hurle et al. [26] have experimentally studied a differentially-heated-boat natural convection problem for molten gallium. In [27], Hart has analyzed the stability of shallow Hadley circulating low-*Pr* flows contained in a shallow slot. Winters [28] has carried out studies regarding the *Pr* effect, aspect ratio effect and boundary effect of oscillatory convection in liquid metals in a 2D rectangular enclosure. Kamakura et al. [29] have numerically investigated the oscillatory phenomena in natural convection of low-Pr fluids in a rectangular cavity with horizontal temperature gradient at $Pr = 0.01$. In [30], Crunkleton et al. have numerically computed the transition from steady to oscillatory flow for $Pr = 0.008$ in rectangular enclosures. Hung et al. [31] have reported the detailed experimental study of transitions in the convection of a low-Pr fluid driven by a horizontal temperature gradient. Kamotani et al. [32] have focused on three-dimensional effects of oscillatory convections development in rectangular enclosure filled mercury fluids for $Gr > 10^7$. Wakitani et al. [33] have numerically investigated dependence of the onset of oscillation on *Pr*'s, the aspect ratio (*AR*), and the width ratio (*WR*) for 3*D* natural convection at low *Pr*'s from 0 to 0.027 in rectangular enclosures with differentially heated vertical walls. Afrid and Zebib [34] have obtained numerical results for a zero-*Pr* fluid in an enclosure with $AR = 4$, and $WR = 1$ or 2. They have established the pattern of three-dimensional convection and determined the critical value of the Grashof number for the onset of oscillation. Vedantam et al. [35] have investigated the effect of two large *AR*'s (5 and 10) on low-*Pr* fluids oscillatory behaviors of the *Nu* for 3*D* Rayleigh-Bénard convection in a rectangular enclosure. Besides, a small number of investigators have focused on bifurcations and chaos [36–39] for low-*Pr* fluids.

In the present work at $Pr = 0.025$, we have discovered cascade-like behaviors of the $\overline{Nu_{avg}}$ in the *AR* domain and cyclic characteristics of the peak of *Nu* in the time domain. For the former, we have managed to identify four regimes and the associated mechanism. For the latter, we have summarized these intriguing phenomena and share this summary with readers. At the same time, the proposed study serves as the sequel of Ref. [1], thus completing a comprehensive aspect-ratio-related study of buoyancy-driven recirculating flows for the entire range of *Pr*.

2. Numerical model

Consider buoyancy-driven recirculating flows for low-*Pr*-number fluids (with thermophysical properties listed in Table 1 for mercury) shown in Fig. 1(a), accompanied by a typical finite-element grid (Fig. 1(b)).

2.1. Governing equations as well as associated initial and boundary conditions

Continuity, Navier-Stokes momenta, and energy transport are governed by

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho u v}{\partial y} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v u}{\partial x} + \frac{\partial \rho v^2}{\partial y} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \rho g, \tag{3}$$

$$\frac{\partial \rho T}{\partial t} + \frac{\partial \rho u T}{\partial x} + \frac{\partial \rho v T}{\partial y} = \frac{k}{c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

and the equation of state

Table 1
Thermophysical properties of mercury at 20 °C.

Properties	Mercury
c_p (J/kg K)	139.52
ρ (kg/m^3)	13580.75
$\mu \times 10^3$ (kg/(m s))	1.55
k (W/m K)	8.698
$\beta \times 10^4$ (1/K)	1.803
$\alpha \times 10^7$ (m^2/s)	45.905
Pr	0.025