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# Triple diffusive mixed convection from an exponentially decreasing mainstream velocity

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ABSTRACT

Current paper deals with the numerical study on steady triple diffusive mixed convection boundary layer flow for exponentially decreasing external flow velocity in presence of suction/injection. Such exponentially decreasing external flows have specific applications in diverging channel flows. The temperature of the vertical surface is assumed to be higher compared to the surrounded fluid temperature. In the triple diffusive flow, the solutal components are chosen as Sodium chloride and Sucrose and the components are added in the flow stream from below with various concentration levels. The concentrations of NaCl-Water and Sucrose-Water are assumed to be lower within the free stream compared to the species concentrations of NaCl-Water and Sucrose-Water near the wall. The coupled nonlinear partial differential equations governing the flow, thermal and species concentration fields are transformed using the non-similarity variables and solved numerically by an implicit finite difference scheme with quasi-linearization technique. The effects of wall suction/injection, Richardson number, decelerating parameter, ratio of buoyancy parameters and Schmidt numbers of both the solutal components on the fluid flow, thermal and species concentration fields are analyzed and discussed. Results indicate that the thickness of the momentum boundary layer is lower for suction compared to injection for the buoyancy opposing flow. The decelerating parameter has significant impact on the flow fields. Also, the species concentration boundary layer thickness decreases with the increase of Schmidt numbers and that increases with the ratio of buoyancy parameters for both the species components. Overall, the mass transfer rate is found to increase with Schmidt numbers approximately 10% and 64% for NaCl and Sucrose, respectively.


## 1. Introduction

Since last few decades' research has been carried out on mixed convection boundary layer flow involving heat and mass transfer due to its various applications in engineering and science. In particular, the significant investigation has been done to follow the characteristics of the fluid flow, heat and mass transfer in the presence of suction or injection on natural and mixed convection boundary layer flows. The boundary layer thickness grows due to the presence of the adverse pressure gradient and after that breaks away from the bounding wall. In other words, boundary layer flow separation occurs. Literature review shows that the development of the boundary layer and the point of separation strongly depends on the pressure distribution. Riley and Stewartson [1] carried out a comprehensive analysis on trailing-edge flows where the free stream velocity of the boundary layer flow was considered as $u_e = u_0(1-\xi)^{\epsilon}$ where $u_0$ is constant, $\epsilon$ is a small parameter $(0 < \epsilon \ll 1)$ and $\xi$ is a scaled streamwise coordinate. Curle [2] conducted an analytical study on the development of a steady incompressible laminar boundary layer flow with the external flow velocity $u_e = u_0(1 - \epsilon e^{\xi})$. It was revealed from the study that for the smaller values of both $\epsilon$ and $\xi$, the contribution of the term $\epsilon e^{\xi}$ in the flow fields is not significant. In contrast, the effect of $\epsilon e^{\xi}$ becomes prominent with the increase $\xi$ and as $\xi$ approaches to $log(\epsilon^{-1})$ the boundary layer separation occurs. Chiam [3] numerically studied the effect of uniform suction on the steady laminar boundary layer flow with the exponentially decreasing free stream velocity similar to the work done by Curle [2]. It was concluded that the exponentially decreasing velocity has significant effect on the flow fields and the delay in separation occurs due to the

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**Nomenclature**

| | |
|---|---|
| $C$ | species concentration |
| $C_f$ | local skin friction coefficient |
| $C_p$ | specific heat capacity |
| $D$ | mass diffusivity |
| $f$ | dimensionless streamfunction |
| $g$ | acceleration due to gravity, m s$^{-2}$ |
| $G$ | dimensionless concentration for NaCl |
| $Gr$ | Grashof number |
| $H$ | dimensionless concentration for Sucrose |
| $k$ | thermal conductivity, W m$^{-1}$K$^{-1}$ |
| $L$ | characteristic length, m |
| $Nu$ | local Nusselt number |
| $Pr$ | Prandtl number |
| $Re$ | Reynolds number |
| $Ri$ | Richardson number |
| $Sc$ | Schmidt number |
| $Sh$ | Sherwood number |
| $T$ | temperature of the fluid, K |
| $u$ | $x$ component of velocity, m s$^{-1}$ |
| $U$ | composite reference velocity, m s$^{-1}$ |
| $U_w$ | moving plate velocity, m s$^{-1}$ |
| $U_\infty$ | free stream velocity, m s$^{-1}$ |
| $v$ | $y$ component of velocity, m s$^{-1}$ |
| $x$ | distance along $x$ coordinate, m |
| $y$ | distance along $y$ coordinate, m |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity, m$^2$ s$^{-1}$ |
| $\beta_T$ | volume expansion coefficient, K$^{-1}$ |
| $\beta_c$ | solutal expansion coefficient, mol liter$^{-1}$ |
| $\theta$ | dimensionless temperature |
| $\nu$ | kinematic viscosity, m$^2$ s$^{-1}$ |
| $\rho$ | density, kg m$^{-3}$ |
| $\psi$ | dimensionless streamfunction |
| $\mu$ | dynamic viscosity, kg m$^{-1}$ s$^{-1}$ |
| $\xi, \eta$ | transformed variables |

*Subscripts*

| | |
|---|---|
| $c_1, c_2$ | first and second species components, respectively |
| $w, \infty$ | conditions at the wall and infinity, respectively |
| $\xi, \eta$ | partial derivatives w.r.t. these variables, respectively |

presence of uniform suction. Roy and Saikrishnan [4] studied the effect of multiple slot suction or injection in a laminar boundary layer flow with the exponentially decreasing free stream velocity. In addition, Roy et al. [5] studied the influence of non-uniform slot suction or injection for the boundary layer flow in a diverging channel involving exponentially decreasing free stream velocity. Recently, Patil et al. [6] analyzed the heat and mass transfer characteristics during mixed convection for exponentially decreasing free stream velocity. Patil et al. [7] further extended the earlier work [6] for unsteady case with similar boundary conditions.

The study of the boundary layer flow separation and the control of separation have relevant technological applications [8–11]. Suction or injection through the boundary wall is found to be one of the conventional technique to control the flow separation. Raptis et al. [12] investigated double-diffusive free convection boundary layer flow through a porous medium in the presence of suction. Also, Raptis and Tzivanidis [13] extended their study for the unsteady case. The effect of flow injection on the fluid, heat, and mass flow fields during natural convection along vertical plate embedded by porous media was studied by Lai and Kulacki [10]. Bansod [11] examined the effect of suction and injection on the flow characteristics during double-diffusive mixed convection over an inclined surface. Mahdy [14] dealt with the Soret and Dufour effect on the heat and mass flow fields during mixed convection from a vertical surface in the presence of suction and injection. Bhattacharyya and Layek [15] have analyzed the effect of suction or injection on boundary layer stagnation-point fluid flow and heat transfer around a shrinking sheet in the presence of thermal radiation. Lin et al. [16] studied the combined effect of magnetic field and suction or injection on the heat and mass transfer of power-law fluid past a horizontal surface.

Although several studies have been done to analyze the effect of suction or injection on the double-diffusive mixed convection boundary layer flow [17], no attempt has been done for triple diffusive mixed convection flow. In case of triple diffusive boundary layer flow, the change of density occurs based on the thermal diffusivity and the solutal diffusivities of two different chemical species. Recently, attention has been focused on the study on triple diffusive convection due to its several applications in science. Patil et al. [18] have investigated the triple diffusive mixed convection effects on vertically moving surface. Khan et al. [19] and Khan et al. [20] have studied the triple diffusive boundary layer natural convection flow from a horizontal plate in the presence of nanofluid saturated in porous media and pure fluid saturated in porous media, respectively. Shivakumara and Naveen Kumar [21] dealt with the influence of couple stresses on the linear and weakly nonlinear stability of a fluid layer during triple diffusive natural convection flows. Linear stability analysis of a triply diffusive fluid layer rotating vertically with a uniform velocity was carried out by Prakash et al. [22]. Thus, the literature review shows that attempts are required to study the effect of suction or injection on the triple diffusive mixed convection boundary layer flow which has direct applications in food processing [23,24].

The main objective of the current study is to analyze the fluid flow, heat and mass transfer on a laminar boundary layer flow during triple diffusive mixed convection in presence of the exponentially decreasing free stream velocity. The effects of suction or injection on the flow fields are also analyze and discussed. The set of governing equations are solved using implicit finite difference scheme in combination with the Quasi-linearization technique to obtain the non-similar solutions. The numerical results are presented in terms of velocity, temperature and concentration profiles for various parameters. In addition, the rate of heat and mass transfer for various chemical species are presented via gradient profiles.

## 2. Governing equations, boundary conditions and numerical simulations

The physical domain consists of a steady laminar mixed convection boundary layer flow of a semi-infinite vertical surface embedded by fluid with exponentially decreasing external flow velocity [Fig. 1]. In the computational domain, the plate is kept vertically along the $x$-axis and the $y$-axis is taken perpendicular to it. It is assumed that the fluid is flowing with an exponentially decreasing velocity $(U_\infty(1 - \epsilon e^{\xi}))$ in the upward direction [4] where $\epsilon$ is known as the decelerating parameter $(0 \leqslant \epsilon \leqslant 1)$. The temperature of the wall $(T_w)$ is assumed to be constant. The concentration of