International Journal of Heat and Mass Transfer 124 (2018) 454–462

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

New friction factor and Nusselt number equations for turbulent convection of liquids with variable properties in circular tubes

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ARTICLE INFO

Article history:
Received 1 January 2018
Received in revised form 13 March 2018
Accepted 24 March 2018

Keywords:
Turbulent convection
Friction factor
Nusselt number
Variable property
Law of the wall
Prandtl mixing length theory
Turbulent boundary layer theory

ABSTRACT

Large temperature differences between the wall and bulk fluid in heat exchangers will result in significant property variations in the cross section. Most existing equations use correction factors such as $(\mu_w/\mu_b)^n$ or $(Pr_w/Pr_b)^m$ to take account of property variation effects on heat transfer coefficients and friction factors. The exponents, n or m obtained by regression analysis of experimental data are not consistent in the literature. They are also different for heating and cooling conditions. In the current investigation, velocity and temperature distributions of turbulent convection with variable properties in circular tubes are analyzed using classical turbulent boundary layer theory. The viscosity variation effects on velocity and temperature fields are simplified to the effects on the critical point between the linear and the logarithmic distribution regions. New friction factor and Nusselt number equations for turbulent convection of liquids with variable properties are obtained. The new equations show accurate predictions of experimental data for both heating and cooling conditions and for different kinds of liquids. The new equations show that viscosity variation effects on friction factor decrease with the increasing of Reynolds number, while its effects on Nusselt number almost keep constant with the increasing of Reynolds number.

1. Introduction

Most conventional friction factor and Nusselt number equations for turbulent convection are obtained on the basis of constant property assumptions, like the Prandtl-Kármán's implicit friction factor equation (Eq. (1)) and Kays's Nusselt number equation (Eq. (2)) [1].

$$\sqrt{1/f} = 2.0\log(Re\sqrt{f}) - 0.8 \tag{1}$$

$$Nu = \frac{RePr\sqrt{f/8}}{0.88 + 13.39(Pr^{2/3} - 0.78)\sqrt{f/8}} \tag{2}$$

However, temperature differences between hot and cold working fluids of heat exchangers in chemical engineering and nuclear industries are usually very large [2,3]. The large temperature differences between the wall and the fluid will result in large properties variation in the cross section. Friction factors and Nusselt numbers calculated using the constant property equations will deviate significantly from experimental data [4]. For liquid flow with heating conditions, the wall shear stress decreases and velocity increases near the wall region due to the decrease in dynamic viscosity with the increase in temperature. As a result, friction factors decrease and heat transfer coefficients increase. Property variation effects on turbulent convection of liquid have been theoretically, numerically and experimentally investigated.

Hanna and Sandall [5] analyzed viscosity variation effects on turbulent water flow in straight tubes. They developed a heat transfer coefficient equation composed of viscosity-temperature sensitivity parameter, friction factor ratio of variable to constant property convections and temperature ratio of bulk fluid to the wall. Thomas et al. [6] also derived a heat transfer coefficient equation for turbulent convection with variable viscosity at moderate Prandtl numbers. Friction factor in both of these two equations [5,6] are calculated using Petuhov's [7] equation, which was obtained by regression analysis of experimental data. Zhang and Cao [8] investigated the property variations effects on natural convection using Lattice Boltzmann method. Ibrahim and Walker [9] and Chyouv and Sleicher [10] modified the constant property eddy diffusivity model to simulate turbulent convection with variable properties. Variation effects of specific heat, thermal conductivity, density and dynamic viscosity on heat transfer and fluid flow were

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https://doi.org/10.1016/j.ijheatmasstransfer.2018.03.082
0017-9310/

Nomenclature

Notation

c_p	specific heat capacity (J kg^{-1} K^{-1})
DNS	direct numerical simulation
f	friction factors, $f = 8\tau_w/\rho u_a^2$
Nu	Nusselt number (dimensionless)
r_0	tube radius (m)
Pr	Prandtl number (dimensionless)
Re	Reynolds number (dimensionless)
T	temperature (K)
u	velocity magnitude in the axial direction (m s^{-1})
V^*	friction velocity magnitude $V^* = \sqrt{\tau_w/\rho}$ (m s^{-1})
x	parameter for the referenced temperature calculation
y	distance from the wall (m)

Greek letters

ε	eddy viscosity (m^2 s^{-1})
κ	Kármán constant, 0.41
μ	dynamic viscosity (kg s^{-1} m^{-1})
ρ	density (kg m^{-3})
τ	shear stress (Pa)
v	kinetic viscosity (m^2 s^{-1})

Subscript

0	center
a	average
b	bulk
c	constant property
e	referenced temperature
$crit$	critical point
f	film temperature $T_f = (T_w + T_b)/2$
l	local
m	momentum
max	maximum
t	turbulent
v	variable property
w	wall

separately investigated. DNS (direct numerical simulation) method was also used to investigate property variation effects on flow and temperature fields in the boundary layer [11–13]. Lee et al. [14,15] investigated viscosity variation effects on turbulent thermal boundary layer over isothermally heated walls using DNS method. The wall stresses decreased and turbulence energy increased due to the lower viscosities near the wall region. The near wall velocity fluctuations and wall-normal heat flux were higher than those in constant viscosity flows. The friction factors decreased and heat transfer coefficients increased with the increasing of wall temperatures. New equation for temperature distribution in thermal boundary layer was obtained to take account of viscosity variation effects. Convection of supercritical fluids is always accompanied with significant property variations, which has attracted many investigations [16,17]. Bae and Yoo [18] and Nemati et al. [19] simulated turbulent convection of CO$_2$ at supercritical pressures with DNS. The thermal expansion effects and buoyancy effects on flow and thermal fields were analyzed with turbulent statistics. The significantly changing properties of supercritical fluids include density, thermal conductivity and heat capacity other than viscosity, which is more complicated.

In the literature, many friction factor and Nusselt number equations have been presented utilizing the constant property assumptions. For example, the friction factor and Nusselt number equations proposed by Petukhov [7] can been used for large range of Reynolds numbers and Prandtl numbers. Investigating the relationship between pressure drops and heat transfer performances, Everts and Meyer proposed new Nusselt number equation for large range of Reynolds numbers [20]. To take account of property variation effects, most empirical friction factor and heat transfer coefficient equations are obtained by regression analysis of experimental data based on two methods. One is the referenced temperature method shown in Eq. (3). By using this method, properties in the constant property equations are calculated by referenced temperature, T_e, to take account of the property variation effects. Deissler [21] simulated turbulent convection with variable viscosities in straight tubes and found that the parameter of x is a function of Prandtl number. On the basis of the numerical results, Deissler proposed a plot to calculate the effective referenced temperature for friction factor and proposed another one for heat transfer coefficients.

$$T_e = x(T_w - T_b) + T_b \tag{3}$$

Ghajar and Parker [22] used the referenced temperature method to calculate Nusselt number for laminar free convection on vertical flat plates at supercritical pressures. Plots for liquids of Refrigerant-114, water and carbon dioxide were presented to determine the parameter of x in Eq. (3). Some researchers also suggested that properties in the conventional equation should be calculated by different temperatures, which is similar to the referenced temperature. Sleicher and Rouse [23] proposed to use the film temperature, $T_f = (T_b + T_w)/2$, to calculate viscosity and use wall temperature to calculate the Prandtl number.

The other method is the property ratio correction method, which is more popular due to the simplicity. In this method, friction factors and Nusselt numbers calculated by the constant property equations are modified by the viscosity ratios, which are shown in Eqs. (4) and (5).

$$\frac{Nu_v}{Nu_c} = \left(\frac{\mu_w}{\mu_b}\right)^n \tag{4}$$

$$\frac{f_v}{f_c} = \left(\frac{\mu_w}{\mu_b}\right)^m \tag{5}$$

By regression analysis of experimental data, different researchers suggested different specific values of n and m. Seider and Tate [24] suggested that n is -0.14 and m is 0.14 for both heating and cooling conditions. Petukhov [7] proposed that n is -0.11 for heating condition and n is -0.25 for cooling condition. Petukhov also suggested m to be 0.24 for cooling condition and proposed a linear function of μ_b/μ_w to calculate the friction factor ratio with heating condition. Allen and Eckert [25] and Choi and Cho [26] suggested m to be 0.25. Büyükalaca and Jackson [27] found that n is a function of Reynolds number. Gnielinski [28,29] proposed to use $(Pr_w/Pr_b)^{-0.11}$ to calculate the property variation effects on Nusselt number ratio. It is similar to the form of $(\mu_w/\mu_b)^{-0.11}$ in the case of turbulent convection of liquid. This is because the variation of specific heat and thermal conductivity can be neglected compared with viscosity for liquid flow. Previous investigations showed that different researchers proposed different exponents. Previous researchers also used different constant property equations to cal-