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Similarity-solution-based improvement of γ-$Re_{\theta t}$ model for hypersonic transition prediction

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ABSTRACT

The improvement of γ-$Re_{\theta t}$ model for hypersonic transition prediction is conducted based on the compressible similarity solutions. The Wilcox's correlation of vorticity Reynolds number and momentum thickness Reynolds number adopted in the original γ-$Re_{\theta t}$ model is not suitable for hypersonic boundary layer. The new correlation is obtained from similarity solutions of compressible boundary layer equations, which includes parameters such as Mach number and temperature of boundary edge and wall temperature. Then the new correlation as well as several modifications are applied to improve the γ-$Re_{\theta t}$ model for hypersonic transition prediction. Four test cases are selected to assess the performance of the improved γ-$Re_{\theta t}$ model, including a wide range flows from two-dimensional flat plate and double ramp to three-dimensional X-51A forebody and scramjet intake. The predicted pressure coefficient and Stanton number are consistent with the available experimental data, which validate the transition prediction capacity of the improved γ-$Re_{\theta t}$ model in different hypersonic conditions. For complex scramjet intake, the predicted results by the improved γ-$Re_{\theta t}$ model show a good agreement with experimental data, especially in the interior region, which demonstrates that the improved γ-$Re_{\theta t}$ model can be an effective tool for the design and optimization of hypersonic vehicles.

1. Introduction

Boundary layer transition is the process of flow transiting from laminar to turbulent, which plays a crucial role in the design and optimization of hypersonic vehicles, as the heat transfer and skin friction of turbulence are much higher than those of laminar flow. Accurate prediction of transition location and length can help to optimize thermal protection systems, which will greatly decrease the gross weight of vehicles.

Up to now, different kinds of numerical methods for boundary layer transition prediction have been developed, such as semi-empirical e^N method, direct numerical simulation (DNS), large eddy simulation (LES), and transition models. The semi-empirical e^N method, based on the linear stability analysis, has been successfully applied to predict natural transition. While in the application of the e^N method, high-precision mean-flow data is required, but for the hypersonic flow, it is still not clear whether entropy layer, viscous interaction, and so on are solved with sufficient accuracy. What's more, it is difficult to predict transition of complex three-dimensional flow. DNS and LES are suitable methods for transition prediction, which can help to explore the mechanism of receptivity, non-linear regime and turbulent spots [1]. But a large amount of computing resources are necessary for DNS and LES, which restricts their applications in engineering problems. The transition model based on Reynolds averaged Navier–Stokes (RANS) equations is a compromise between the accuracy and cost of computation. In last decades, more than ten kinds of transition models have been developed. Nevertheless, the physical transition mechanisms are different in various flows, such as natural, bypass, separation, and crossflow and Göertler instability. It is challenging to develop a universal transition model, which is valid for all of those different mechanisms. Even though, transition models have been widely applied in engineering design, and a lot of significant results can be found in open literatures [2,3].

Since the intermittency factor γ was firstly proposed by Dhawan and Narasimha [4] in 1958, various kinds of transition models have been developed based on the intermittency concept. Steelant and Dick [5] derived a transport equation of γ based on the correlation suggested by Dhawan and Narasimha [4]. Cho and Chung [6] proposed a k–ε–γ model based on the works of Libby [7], which was intended to simulate the transition behavior in free shear flows. Suzen and Huang [8,9] developed a new transport equation for intermittency factor γ, which combined the properties of the two

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models mentioned above. The most prominent property of this model is that it is able to reproduce the distribution of intermittency factor γ in both streamwise and normal directions. Based on the results of linear stability theory (LST), Warren and Hassan [10,11] proposed a new k–ε–γ transition model aimed at prediction of hypersonic boundary layer transition. The concept of effective eddy viscosity μ_{eff} was first proposed in the model, which was the sum of turbulent viscosity μ_t and non-turbulent viscosity μ_{nt} with intermittency-weighted. The non-turbulent viscosity μ_{nt} was computed through timescales of first and second mode disturbances. Based on the μ_{eff} concept, Papp and Dash [12,13] added laminar kinetic energy k_L and γ equations to the SSGZ k-ε model to develop the SSGZ-k_L-γ transition model, which further improved the prediction performance of Warren and Hassan' model [10,11]. It should be noted that, the application of those models mentioned above need computing additional non-local variables, such as boundary layer thickness or momentum thickness, thus they are not compatible with modern CFD codes based on unstructured grids and massive parallel execution.

To avoid computing non-local variables, some new transition models, which were strictly based on local variables, were developed. For example, Walters and Cokljat [14] developed a physics-based k_T-k_L-ω transition model, in which the laminar kinetic energy concept proposed by Mayle and Schulz [15] was introduced to model low-frequency pre-transitional fluctuations. Langtry and Menter [16,17] proposed a new local correlation-based transition model (LCTM), named γ-$Re_{\theta t}$ model, which was strictly based on local variables by adopting the correlation of vorticity Reynolds number Re_v and momentum thickness Reynolds number Re_θ. The correlation of Re_v and Re_θ was first proposed by Wilcox [18], which was obtained from Blasius boundary similarity solutions. Durbin [19] and Ge [20] proposed a simple k–ω–γ model for representing bypass transition. The transition is initiated by diffusion, and a source term combined Re_v and turbulence scale, which is based on the correlation of Praisner and Clark [21], carries it to completion. Wang and Fu [22,23] proposed a k-ω-γ transition model based on the μ_{eff} concept of Warren and Hassan [10,11]. This model is also inspired by the correlation of Re_v and Re_θ proposed by Wilcox [18], and transforms the correlation to the length scale, which is used to calculate the characteristic time scale of different instabilities including first Mack mode, second Mack mode, etc.

Among those transition models based on local variables, the γ-$Re_{\theta t}$ model proposed by Langtry and Menter may be the most popular model, and has been implemented in some commercial software [24]. The model has shown good performance in many subsonic cases, ranging from natural transition to bypass transition, separation-induced transition, and even re-laminarization. In addition, Langtry [25] and Grabe [26] also extended this method to predict crossflow transition. In recent years, many efforts have been made to extend this model to supersonic even hypersonic flows. Krause [27] presented new correlations of $Re_{\theta c}$ and F_{length} based on the experimental data of hypersonic flat plate cases conducted by Mee [28]. The correlations are the function of freestream turbulent intensity Tu_∞ and used to replace the original correlations in Langtry and Menter's model. Based on the Krause's correlations, You [29] took the effect of pressure gradient into consideration, and made further improvement of the γ-$Re_{\theta t}$ model for hypersonic transition prediction. In addition, Frauholz [30] coupled the γ-$Re_{\theta t}$ transport equations with *SSG/LRR* turbulence model with a modified ansatz of Krause's correlations, and the model showed good performance in predicting transition of hypersonic scramjet intake configurations. Recently, Hao [31] proposed a different idea from the previous works. A new correlation of $Re_{v,\max}$ and Re_θ for hypersonic boundary layer was obtained from flat plate CFD simulations using boundary layer parameters identification method [32]. The performance of improved γ-$Re_{\theta t}$ model was validated by experimental results. However, since the correlation is obtained by several typical cases and lacks theoretical basis, the universality somewhat remains to be evaluated.

Inspired by the Wilcox's correlation [18] of $Re_{v,\max}$ and Re_θ generated from Blasius similarity solutions, the new correlation of $Re_{v,\max}$ and Re_θ is obtained from similarity solutions of compressible boundary layer equations. The correlation of $Re_{v,\max}$ and Re_θ in original γ-$Re_{\theta t}$ model is then replaced by the new one to improve the transition prediction capacity for hypersonic flow. The performance of the improved γ-$Re_{\theta t}$ model is validated with a wide range flows from two-dimensional flat plate and double ramp to three-dimensional X-51A forebody and scramjet intake.

2. The original γ-$Re_{\theta t}$ model and numerical methods

2.1. The original γ-$Re_{\theta t}$ model

The original γ-$Re_{\theta t}$ transition model proposed by Langtry and Menter [16,17] is built strictly on local variables. The model is based on the two-equation *SST* turbulence model [33], and two additional transport equations are added to model the transition process. One is the intermittency factor γ equation, which is used to trigger the transition and control transition length. The other is momentum thickness Reynolds number $\tilde{Re}_{\theta t}$ equation, which can include the effects of turbulence intensity and pressure gradient. They are as follows:

$$\frac{\partial(\rho\gamma)}{\partial t}+\frac{\partial(\rho U_j\gamma)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right]+P_\gamma-E_\gamma \tag{1}$$

$$\frac{\partial(\rho\tilde{Re}_{\theta t})}{\partial t}+\frac{\partial(\rho U_j\tilde{Re}_{\theta t})}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu+\mu_t)\frac{\partial\tilde{Re}_{\theta t}}{\partial x_j}\right]+P_{\theta t} \tag{2}$$

The production term P_γ of γ equation is defined as below:

$$P_\gamma=c_{a1}\rho F_{length}S[\gamma F_{onset}]^{0.5}(1-c_{e1}\gamma) \tag{3}$$

where μ_t is the turbulence eddy viscosity, μ is the molecular viscosity, and S is the strain-rate magnitude. The F_{length}, which is used to control the transition length, is an empirical correlation of $\tilde{Re}_{\theta t}$ based on the results of experiments T3B, T3A, T3A-, and the experiments conducted by Schubauer and Klebanof [34]. σ_γ, $\sigma_{\theta t}$, c_{a1}, and c_{e1} are constants in the model, here $\sigma_\gamma = 1.0$, $\sigma_{\theta t} = 2.0$, $c_{a1} = 2.0$, $c_{e1} = 1.0$. F_{onset} is used to control the transition onset and composed of the following functions:

$$\begin{cases} F_{onset1}=\frac{Re_v}{2.193Re_{\theta c}} \\ F_{onset2}=\min\left(\max(F_{onset1},F_{onset1}^4),2.0\right) \\ F_{onset3}=\max\left(1.0-\left(\frac{R_T}{2.5}\right)^3,0\right) \\ F_{onset}=\max(F_{onset2}-F_{onset3},0) \end{cases} \tag{4}$$

In which, R_T is the turbulent Reynolds number and $Re_{\theta c}$ is the critical Reynolds number, computed from $\tilde{Re}_{\theta t}$. Once the local value of Re_v larger than $2.193Re_{\theta c}$, the intermittency then starts to increase in the boundary layer. The correlation of Re_v and Re_θ was proposed by Wilcox [18], and obtained from Blasius boundary similarity solutions, which was only validated for incompressible flows. The destruction E_γ is defined as follows:

$$E_\gamma=c_{a2}\rho\Omega\gamma F_{turb}(c_{e2}\gamma-1) \tag{5}$$

where Ω is the vorticity magnitude. The constants for the E_γ are $c_{a2} = 0.06$, $c_{e2} = 50.0$.

The source term $P_{\theta t}$ of transition momentum thickness Reynolds number transport equation is defined as follows: