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A finite volume method to solve the frost growth using dynamic meshes

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ABSTRACT

The physical mechanisms of frost formation have been widely studied, yet much empirism is still needed in numerical approaches. Indeed, accurate simulations of frost growth can be reached by setting up a specific combination of the model empirical inputs while using a method to accurately track the frost-air interface.

This paper presents a finite volume ALE method which captures the air-frost interface using dynamic meshes. It is divided into two main sections. First, the search of a valid set of empirical correlations to correctly emulate frost growth under certain experimental conditions. An assessment of seven reference cases is carried out by comparing solutions using different empirical correlations against experimental data. As a result, a discussion on the performance of such parameters is made, emphasizing the fact of using diffusion resistance factors above 1.0 in order to capture the frost growth. Second, a 2D numerical test consisting of a duct flow with a non-homogeneously cooled lower boundary is performed. Aspects related to the frost thickness and growth rate are analysed, proving the method to be a valid candidate to simulate frost growth.

1. Introduction

Whenever a surface is in contact with humid air below the dew and freezing points, water vapour will desublimate, transitioning to a solid state that will form a crystaline structure called frost. Frost formation is a common and usually undesired phenomenon that affects the aerospace, cryogenics and refrigeration industry, among others. Frost can form on aircraft wings either on-ground (typically through nocturnal frost), and in-flight (when crossing supersaturated icing clouds [1]), adding weight and reducing the aerodynamical performance. It also causes a great impact on wind turbines, heat exchangers, engine turbine blades, electrical lines, etc. These issues highlight the need of understanding and accurately predicting frost formation.

Hayashi et al. [2] divided the frost formation mechanism into three periods: the *crystal growth period*, the *frost layer growth period* and the *frost layer full growth period*. The first refers to an early growth period characterized by crystal growth. First, heterogeneous nucleation and further embryo growth covers the cold wall as a thin frost layer. In the second period, the frost layer behaves as a porous medium, where crystals continue growing while interacting with each other. In this period, the initial rough frost becomes a uniform layer. It is considered to end when the thickness of the frost stops growing. The third period continues with a densification and growth of the frost layer, bringing with it an increase of the thermal resistance [2]. During that stage, the thickness growth is minimal, and the frost surface temperature rises until the melting point. From that point onwards, new deposition sites at the frost surface will form in liquid phase, which will soak into the frost layer, freezing in the inside. This cycle process continues periodically until the heat transfer condition reaches the equilibrium.

The frost layer growth period is the most studied among the three. Brian et al. [3] proposed first analytical approximations to model frost growth. Later on, Tao et al. [4] and Le Gall et al. [5] used averaged finite volume approaches, which were also used by Na and Webb [6] with some simplifications. Na and Webb's formulation was subsequently used by Lenic et al. [7], and recently by Armengol et al. [8] to address the air-frost coupled problem. Other significant approaches involving the computation of the fluid and frost domains are the coupling of a one dimensional frost model with a commercial CFD code by Ellgas and Pfitzner [9], the one-domain approach by Kim et al. [10], in which the one dimensional frosting model [11] is implemented, and the recent frost formation resistance model put forward by Kim et al. [12].

Despite the efforts to simulate the frost-free air coupled problem, there is still a lack of consensus in which empirical correlations capture better the frost formation. Furthermore, solutions

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Nomenclature

c_p	specific heat capacity, J/(kgK)
D	diffusivity, m^2/s
D_h	hydraulic diameter, m
h	enthalpy, J/kg
h_{sv}	latent heat of sublimation, J/kg
h_c	convective heat transfer coefficient, W/(m^2 K)
h_m	mass transfer coefficient, m/s
$\vec{j}$	diffusion mass flux, kg/(m^2 s)
$\dot{m}$	water vapour mass flux, kg/(m^2 s)
$\dot{q}$	heat flux, W/m^2
$\mathbf{r}$	vector of residuals
S	supersaturation degree
T	temperature, K (Celsius when specified)
P	pressure, Pa
v	velocity, m/s
V	volume, m^3
W	water vapour concentration, kg$_v$/kg$_{da}$
x	mole fraction
$\mathbf{x}$	vector of variables
y	coordinate, m
Y_v	water vapour concentration, kg$_v$/kg$_{ha}$
Greek symbols	
α_r	relaxation factor
δ	relative error
δy	mesh displacement, m
Δy_{fs}	growth displacement, m
ε	porosity
λ	conductivity, W/mK
μ	diffusion resistance factor
ρ	density, kg/m^3
τ	tortuosity
$\dot{\omega}_i$	ice generation, kg/(m^3 s)
$\dot{\omega}_v$	water vapour generation, kg/(m^3 s)
Subscript	
av	averaged
da	dry air
dens	related to the densification of the frost layer
Δy	related to the increase of layer thickness
eff	effective
fl	frost layer (contains ice and humid air)
fs	frost surface
ha	humid air
i	ice
lat	latent
sat	saturated
t	total
v	water vapour
w	wall
x	related to the position in streamwise direction
0	initial value
∞	far field conditions
Superscript	
ℓ	current outer iteration
m	current inner iteration
n	current time step
q	iteration

with CFD approaches use static grids [7,8,13–16], which lead to a non-accurate tracking of the interface.

In view of the reported results, a finite volume approach based on Tao's mathematical formulation, which models the frost layer growth period (until the melting point is reached) using a deformable mesh is here presented. This paper aims to discuss the performance of the tested empirical correlations, and provide new insights throughout a critical analysis of seven reference cases, covering a wide range of experimental conditions. Finally, the stated methodology is used to numerically solve the frosting on a wind tunnel with a non-homogeneously cooled lower boundary, experimentally set up by Kwon et al. [17], in order to test the model capabilities.

2. Physical model and mathematical formulation

The mechanism of frost growth is shown in Fig. 1, in which a local averaged control volume analysis is depicted. The set out approach considers that a volume V is composed by the volumes occupied by the ice V_i, and the humid air V_{ha}. The ice volume fraction or ice porosity is then defined as $\varepsilon_i = V_i/V$. Similarly, the air porosity is expressed as $\varepsilon_{\text{ha}} = V_{\text{ha}}/V$. Moreover, the equality $\varepsilon_{\text{ha}} + \varepsilon_i = 1$ must be preserved. In addition, note that the humid air porosity, the dry air porosity and the water vapour porosity stand for the same volume fraction, i.e., $\varepsilon_{\text{ha}} = \varepsilon_{\text{da}} = \varepsilon_v$. Such porosities will also be represented as ε_v for the sake of clarity.

The assumptions made in the present analysis are: (a) the total gas phase pressure P_{ha} is constant throughout the porous frost layer, and equal to the external atmospheric pressure P_∞; (b) water vapour, dry air and ice are in local thermal equilibrium, i.e. $T_{\text{ha}} = T_v = T_{\text{da}} = T_i$; (c) water vapour inside the frost layer is saturated; (d) the heat and mass transfer analogy is applicable, with a constant Lewis number; (e) convection effects are negligible such that $\vec{v}_{\text{ha}} = 0$ within the frost layer [18], and (f) no movement of the ice crystals is allowed ($\vec{v}_i = 0$).

2.1. The transport diffusion equations

The vapour mass conservation equation reads as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_v}\rho_v \mathrm{d}V_v + \int_{S_v}\rho_v\left(\vec{v}_v - \vec{v}_b\right)\cdot\vec{n}\,\mathrm{d}S_v = \int_V \dot{\omega}_v \mathrm{d}V \tag{1}$$

where the substantial derivative of the Eulerian density field including the volume swept by the mesh equals the generation or destruction of water vapour $\dot{\omega}_v$. Integrating over the volume V and rewriting the equation in terms of the water vapour diffusion mass flux $\vec{j}_v = \rho_v\left(\vec{v}_v - \vec{v}_{ha}\right)$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_v\varepsilon_v \mathrm{d}V + \int_S \rho_v\varepsilon_v\left(\vec{v}_{\text{ha}} - \vec{v}_b\right)\cdot\vec{n}\,\mathrm{d}S + \int_S \varepsilon_v \vec{j}_v \vec{n}\,\mathrm{d}S = \int_V \dot{\omega}_v \mathrm{d}V \tag{2}$$

Applying that $\vec{v}_{ha} = 0$ and introducing Fick's law, i.e. $\vec{j}_v = -\rho_{ha}\tau D_v \nabla Y_v$:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_v\varepsilon_v \mathrm{d}V - \int_S \rho_v\varepsilon_v \vec{v}_b\cdot\vec{n}\,\mathrm{d}S \\ = \int_S \rho_{\text{ha}}\varepsilon_v\tau D_v \nabla Y_v\cdot\vec{n}\,\mathrm{d}S + \int_V \dot{\omega}_v \mathrm{d}V\end{aligned} \tag{3}$$

where Y_v is the concentration of water vapour, and τ is the tortuosity. The effective diffusivity is defined as $D_{\text{eff}} \equiv \varepsilon_v\tau D_v$ or $D_{\text{eff}} \equiv \mu D_v$,