International Journal of Heat and Mass Transfer 123 (2018) 251–260

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A new radial integration polygonal boundary element method for solving heat conduction problems

Miao Cui [a,b], Hai-Feng Peng [a], Bing-Bing Xu [a], Xiao-Wei Gao [a,*], Yuwen Zhang [b,*]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, PR China
[b] Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA

ARTICLE INFO

Article history:
Received 18 November 2017
Received in revised form 23 February 2018
Accepted 26 February 2018

Keywords:
Heat conduction
Boundary element method
Radial integration method (RIM)
Radial integration polygonal boundary element method (RIPBEM)

ABSTRACT

A new approach, radial integration polygonal boundary element method (RIPBEM), for solving heat conduction problems is presented in this paper. The proposed RIPBEM is a new concept in boundary element method (BEM), which would be of great flexibility in mesh generation of complex 3D geometries. Due to the characteristic of arbitrary shapes of polygonal elements, conventional shape functions are insufficient. Moreover, the resulted surface boundary integrals cannot be directly evaluated by the standard Gauss quadrature. To solve these problems, general shape functions for polygonal elements with arbitrary number of nodes are given. To generally and numerically calculate the resulted surface integrals, the radial integration method (RIM) is employed to convert the surface boundary integrals into equivalent contour line integrals of the polygonal elements. As for 3D domain integrals, they are transformed to equivalent line integrals using RIM twice. This methodology can explicitly eliminate strong singularities. Several numerical examples are given to show the effectiveness and the accuracy of the proposed polygonal boundary element method for solving heat conduction problems.

1. Introduction

Heat conduction is a very popular phenomenon that occurs in a solid or static fluid [1], if temperature difference exists [2], and numerical methods [2–8] such as boundary element method [9–14] are frequently used for solving this type of problems. As a numerical and semi-analytical method, boundary element method (BEM) has the advantage of only boundary discretization, and it could reduce the dimensionality of the problem, which has been being paid much attention for solving heat conduction etc. problems [15–18]. In addition, it is very suitable to be employed in inverse problems [19–23], if only boundary physical quantities could be measured. In conventional three-dimensional (3D) BEM, rectangular or triangular boundary elements are used, which restricts the BEM's engineering application to some extent in solving problems with complicated geometries, due to lack of flexibility in mesh generation.

In the present work, a new concept, radial integration polygonal boundary element method (RIPBEM), is proposed for solving heat conduction problems, which could offer great flexibility in mesh generation for complex 3D geometries [24,25]. Polygons are not new concepts, which have been used in finite element method (FEM) [26–28]. However, it has not been introduced or used in BEM, to the best of the authors' knowledge. It is worth emphasizing that this methodology is only for 3D problems, as the boundary of a 2D geometry is a curve or a line, and there is no need to introduce polygons.

Two main challenges are encountered in RIPBEM: One is that the shape functions of polygonal elements should be general, and the other is that the resulted surface boundary integrals over the polygonal elements cannot be directly evaluated using standard Gauss quadrature due to the arbitrary shapes of the polygonal elements. For the first challenge, the general shape functions for polygonal elements with arbitrary numbers of nodes are given. For the second challenge, the radial integration method (RIM) [29] is employed to convert the surface boundary integrals into equivalent line integrals along the contour of the polygonal element. As for the 3D domain integrals coming from the heat generation and variation of the conductivity, they are transformed to equivalent line integrals by using the radial integration method twice. This treatment is an innovation technique and can explicitly eliminate strong singularities appearing in both boundary and domain integrals.

The heat conduction problems are taken as examples to validate the proposed polygonal boundary element method, but the

* Corresponding authors.
E-mail addresses: xwgao@dlut.edu.cn (X.-W. Gao), zhangyu@missouri.edu (Y. Zhang).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.02.111
0017-9310/

Nomenclature

A	coefficients matrix	$\boldsymbol{x}$	vector of coordinates or vector of unknowns
G	Green function	$\boldsymbol{y}$	source point
h	heat convective coefficient, W/(m^2 °C)	$\boldsymbol{z}$	point on boundary
I	identity matrix	$\boldsymbol{z}'$	point on boundary
L	boundary line or length, m		
k	thermal conductivity, W/(m °C)	*Greek symbols*	
N	shape function	α	vector
Ne	number of elements	β	interpolant
$Node$	number of nodes	Γ	boundary
N_α	shape function	Δ	change in variable
$\boldsymbol{n}$	unit outward normal	Ω	domain
$\boldsymbol{n}'$	unit outward normal		
Q	point or heat generation, W/m^3	*Subscripts*	
q	heat flux, W/m^2	b	boundary
R	distance, m	bottom	bottom
r	distance, m	i	internal or the *i*th
S	area, m^2	upper	upper
T	temperature, °C		

method can be straightly employed to solve other engineering problems.

The rest of the paper is organized as follows. In Section 2, we will briefly describe the steady heat conduction problem with heat generation and spatially varying thermal conductivity. In Section 3, the formulation and construction of general shape functions for polygonal boundary elements with arbitrary nodes are introduced. In Section 4, we will describe how the 3D domain integrals are converted into equivalent line integrals in detail. In Section 5, the formation of the system of equations is introduced. In Section 6, several numerical examples are given to validate the effectiveness and the accuracy of the proposed RIPBEM. Finally, concluding remarks are given.

2. Steady heat conduction problem with heat generation and spatially varying thermal conductivity

This paper is to present a new approach, polygonal boundary element method, for solving heat conduction problems, and the steady heat conduction problems with heat generation and spatially varying thermal conductivity are taken as examples. The steady heat conduction problem with heat generation can be expressed as follows:

$$\frac{\partial}{\partial x_i}\left[k(\boldsymbol{x})\frac{\partial T(\boldsymbol{x})}{\partial x_i}\right] + Q(\boldsymbol{x}) = 0 \quad (\boldsymbol{x} \in \Omega) \tag{1}$$

The boundary conditions are:

$$T(\boldsymbol{x}) = \overline{T}(\boldsymbol{x}), \quad \boldsymbol{x} \in \Gamma_T \tag{2}$$

$$q(\boldsymbol{x}) = -k(\boldsymbol{x})\frac{\partial T(\boldsymbol{x})}{\partial \boldsymbol{n}} = \overline{q}(\boldsymbol{x}), \quad \boldsymbol{x} \in \Gamma_q \tag{3}$$

In Eqs. (1)–(3), T is the temperature, °C, k is the thermal conductivity, W/(m °C), i changes from 1 to 3, $\boldsymbol{x} = (x_1, x_2, x_3) = (x, y, z)$. $q(\boldsymbol{x})$ is the normal heat flux on the boundary Γ_q of the computational domain Ω; $\boldsymbol{n}$ is the unit outward normal to Γ_q. In Eqs. (2) and (3), $\overline{T}(\boldsymbol{x})$ and $\overline{q}(\boldsymbol{x})$ are the given temperature and heat flux on the boundary. It should be emphasized that either temperature or heat flux is specified on a boundary node.

3. Shape functions for general polygonal elements with arbitrary number of nodes

To set up polygonal boundary element method for a 3D problem, shape functions for surface boundary polygonal elements with arbitrary number of nodes are essential and of great importance. Due to the complexity of the polygonal elements, there is no unified way to define the interpolation functions on polygonal elements [28]. In BEM, shape functions are based on intrinsic or isoparametric coordinates. Therefore, the Wachspress shape functions [25] are selected for convex polygonal boundary elements expressed in isoparametric coordinates in the present work.

Fig. 1 shows a polygon Ω in isoparametric coordinates. The number of the nodes of the polygon is *Node*. At any point $Q = (Q_\xi, Q_\eta)$ inside Ω or on its boundary Γ, the Wachspress shape function $N_i(Q)$, $i = 1, 2, \cdots, Node$ is defined as

$$N_i(Q) = \frac{\beta_i(Q)}{\sum_{j=1}^{N}\beta_j(Q)} \tag{4}$$

where the interpolant β_i is as follows.

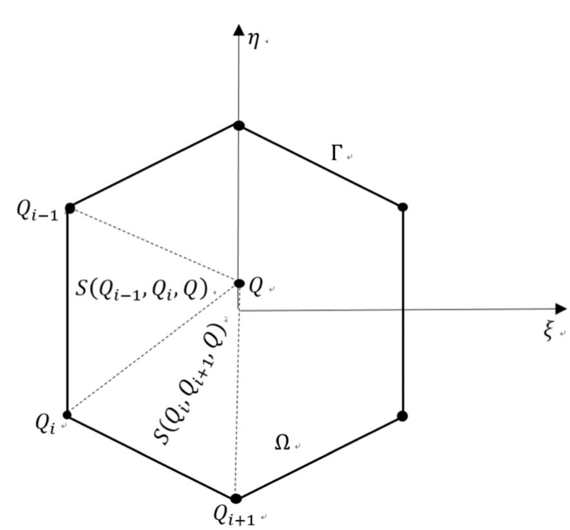

Fig. 1. Arbitrary isoperimetric polygonal boundary element.