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Direct Numerical Simulation of an air-filled differentially heated square cavity with Rayleigh numbers up to 10^{11}

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A set of Direct Numerical Simulations in a heated square cavity invoking the Boussinesq approximation was carried out at Rayleigh numbers ranging between 10^8 and 10^{11} and Prandtl number of 0.71. The three dimensional configurations studied represent an infinitely deep cavity, thus corresponding to a statistically two-dimensional flow with an imposed temperature varying linearly on the horizontal walls. In such configuration, the Rayleigh number, and therefore turbulence intensity, is the highest ever reached. The database presented herein includes first and second order statistical moments as well as full Reynolds stresses, turbulent heat fluxes and temperature variance budgets. The latter are extremely rare for buoyancy driven flow configurations and are therefore believed to be valuable to the turbulence modelling community. The analysis of the data collected thus focuses on aspects of relevance to the Reynolds averaged modelling of such flows. The effect of increasing the Rayleigh number on the flow statistics, Nusselt number predictions and thermal stratification is investigated. The most important aspect influencing the behaviour of the budgets was found to be the displacement of the position of the maximum of temperature variance towards the inner zone of the boundary layer. Such difference in behaviour between the thermal and velocity boundary layers introduces regions of negative production in the budgets that tend to increase with the Rayleigh number. The production of turbulence by buoyancy is also found to be of the same order of magnitude as other budget terms at all Rayleigh numbers.

1. Introduction

Buoyancy-driven flows are ubiquitous in the environment as well as in many industrial configurations. Common examples of such industrial applications include double-glazed windows, passive cooling systems for electronic components and possibly nuclear reactor emergency cooling systems. Rectangular differentially heated cavities have the advantage to be very simple from a geometrical point of view whilst embodying complex physics of interest: spatially developing buoyant boundary layers in the presence of thermal stratification. The present paper focuses on cavities where the main temperature difference is applied between the vertical walls, yielding a mean horizontal gradient of temperature between the walls. This case is different from the Rayleigh-Bénard type convection where the temperature gradient is aligned with the gravity vector in an unstable manner. The purpose of the present work is to provide an analysis of the physics of buoyancy driven flow in a differentially heated cavity with a Reynolds averaged modelling perspective. Rectangular vertical differentially heated cavities have been a topic of interest for decades now. The work of Batchelor [1], who studied analytically the limiting case of very tall cavities at low Rayleigh number, was the first on the topic. Other early studies such as the pioneering work by Elder [2,3] on high aspect ratio cavities were mainly experimental and focused on the identification of the different flow regimes obtained when varying the Rayleigh and Prandtl number. Three flow regimes were identified experimentally: steady laminar flow, unsteady laminar flow and fully turbulent flow. A large body of work has been focused on the characterisation of such flow regimes as a function of the dimensionless numbers governing the flow. When the fluid satisfies the Boussinesq approximation, a simple normalisation of each term of the Navier-Stokes equation shows that the flow is fully characterised by the Rayleigh number, the Prandtl number and the aspect ratio of the cavity. Due to the limited computational resources, numerical solutions of differentially heated cavity flows were originally limited to very low Rayleigh number well within the laminar regime. These early calculations were two-dimensional, usually based on the streamfunction-vorticity formulation of the Navier-Stokes

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equations and relying on the Boussinesq approximation to account for buoyancy effects on the flow [4,5]. Most of the numerical results obtained in the laminar regime were consistent with each other. For instance, the results obtained by De Vahl Davis [6] using Richardson's extrapolation for a square cavity with Rayleigh number between 10^3 and 10^6 are still widely used for code validation purposes. Other studies such as the work of Chenoweth and Paolucci [7] studied the behaviour of the flow in cavities of various aspect ratio with variable fluid properties. They observed that the flow loses its centro-symmetry when departing from a Boussinesq-type fluid and proposed flow regime maps. Much attention in the literature has been paid to the identification of the mechanisms triggering the transition to unsteadiness and later to turbulence. This was initially done by studying the stability of two-dimensional solution of the Navier-Stokes equation [8,9] when introducing small perturbations. It was later shown by Henkes and Le Quéré [10] that the critical Rayleigh number was lower when considering a three-dimensional cavity with periodic boundary conditions in the third direction and applying a three-dimensional perturbation. The critical Rayleigh number was found to depend on the aspect ratio of the cavity [8] and also on the boundary conditions applied to the horizontal walls. Two classic extreme configurations can be found in the literature: adiabatic horizontal wall and perfectly conducting ones (linear variation of temperature between hot and cold side of the cavity), with the former representing the vast majority. Henkes and Le Quéré [10] showed that the critical Rayleigh number for the conducting square cavity is around 2×10^6 whereas the more recent study of Xin and Le Quéré [11] showed that the critical Rayleigh number in an adiabatic three-dimensional square cavity is around 1.55×10^7, which is more than one order of magnitude lower than the critical Rayleigh number observed by the same authors in the two-dimensional case. As regards the fully turbulent flow regime, numerical analysis by Direct Numerical Simulation (DNS) has been impossible for a very long time because of the limited computational resources available and the particular complexity of the problem. Indeed, differentially heated cavity flows often have, in their fully turbulent regime, a quiescent stratified core and very thin buoyant boundary layers along the walls, which demand large computational resources. A very large number of early studies considered the two-dimensional form of the Navier-Stokes equation in the fully turbulent regime [12,9]. It is only in the last decade that accurate solution of the three dimensional problem emerged, firstly with the publication of increasingly resolved large-eddy simulations (LES) [13,14], supported by the publication of experimental data [15–17] followed by direct numerical simulations. A large number of direct numerical simulations in tall differentially heated cavities (aspect ratios between 4 and 5) with increasing Rayleigh number were published in the last decade by the same group of researchers using mostly a fourth order finite volume scheme [18–22]. The square cavity remains a rather rarely tackled case in the literature because of its additional computational cost associated with the higher Rayleigh numbers for the transition to turbulence to occur compared with tall cavities. The combination of these aspects makes the simulation of square cavities at Rayleigh number in the fully turbulent regime very difficult to achieve. Even nowadays, many publications on such configuration are limited to LES results [23–25] or two-dimensional DNS. Sergent et al. [26] used Chebyshev collocation to carry out DNS of a fully three-dimensional box of square cross section with no slip walls on each of the six faces with aspect ratio 0.2 for the depth of the cavity. Puragliesi and Leriche [27] studied the flow in differentially heated cubic cavity at Rayleigh number of 10^9. To the authors' knowledge, this is the highest Rayleigh number ever considered for a true three-dimensional Direct Numerical Simulation for a differentially heated cavity of unit aspect ratio.

From an engineering standpoint, Computational Fluid Dynamics approaches relying on Reynolds Averaged Navier Stokes (RANS) equations are normally used. Application of standard turbulence models such as the $k - \varepsilon$ model associated with standard wall functions was made in differentially heated cavity some time ago [28]. However, numerous studies over the years have shown that application of turbulence models that were initially developed for forced convection flows is inappropriate due to the specific aspects of buoyancy driven flows such as the coexistence of laminar and turbulent regions and the presence of large scale structures. A review of the limitations of single-point mathematical closure when simulating buoyancy driven flow was carried out by Hanjalic [29]. The recent developments of second-moment closure approaches [30–32] focusing on buoyancy driven flows still relies heavily on very simple configurations such as mixed convection in a channel and vertical differentially heated channels [33–35], where the very important turbulent budget data are available at only fairly low Rayleigh number. Very few examples of turbulent budget calculations are presented in publications dealing with the more complicated case of rectangular differentially heated cavities. To the authors' knowledge, Barhaghi and Davidson [36] are the only ones to present the full budgets using LES, making the results only qualitative as mentioned by the authors themselves.

In the present paper, Direct Numerical Simulation results for a Prandtl number equal to 0.71 and Rayleigh numbers varying between 10^8 and 10^{11} in a differentially heated square cavity with prescribed temperature on the horizontal walls are presented. In addition to the first and second order statistical moments, the full budgets of each component of the Reynolds stress tensor are presented as well as the budgets of the turbulent heat fluxes and temperature variance. The decision to focus on the square configuration was motivated by the fact that historically the application of classic turbulence models to this configuration resulted in the largest degree of discrepancy with experiments. In such configuration the two vertical buoyant boundary layers do not interact with each other as they can do in taller cavities via shear in the core of the domain, which makes its modelling more challenging for single point closure turbulence models. Unlike the more recent literature on the topic [37–39] that tends to focus on the coupling between conduction through the walls of the cavities, radiative effects and even particle transport in the fluid, at fairly low Rayleigh numbers, the present work focuses on reaching high Rayleigh numbers to provide important insights from a turbulence modelling standpoint. The analysis presented herein focuses on data of relevance to the RANS modelling framework and does not provide a quantitative analysis of low frequency unsteadiness inherent to such flows. Despite a significant dwindling of academic research focusing on RANS models in aid of LES or hybrid approaches, these models will remain the industry standard for the foreseeable future. It is therefore very important to provide accurate data relevant to this modelling framework that can be used to validate and improve the latest developments in field (see [40,31,41] for instance). The data presented herein and available on the ERCOFTAC classic database are therefore expected to be a valuable resource for the turbulence modeling community as such data are very scarce for buoyancy driven flow configurations other than the simple vertical differentially heated channel.

The paper is organised as follows. In Section 2, the computational methodology will be presented as well as the flow configurations. Then, in Section 3, the influence of the Rayleigh number on the flow with a prescribed linear temperature variation on the horizontal wall is investigated. The latter includes comparison of first order moments with scaling laws, analysis of the evolution