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A new model to determine the thermal conductivity of fine-grained soils

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ABSTRACT

In the study, a three-parameter model was presented to calculate the thermal conductivity at full range of degree of saturation (*S*) for fine-grained soils based on the Fredlund and Xing model and the normalized thermal conductivity method. Three parameters (*a*, *b*, *c*) of the new model are determined by two equations and a measured point at a certain *S*. Two equations were obtained by the correlations between the parameters *a*, *b*, *c* and the basic properties of 30 Canadian soils by regression analysis. Moreover, the relationship between the thermal conductivity and *S* at full range of *S* was defined as the thermal conductivity curve (TCC) and was divided into 3 regions for the improvement of the calculation result. According to the sensitivity analysis, it is found that the calculated TCC is the most reliable when a point is measured in the Region 2. In addition, the new calculation model was also verified by 6 Chinese soils, suggesting the new model could present a good calculation result ($R^2 = 0.97$) for the TCC.

1. Introduction

Soil is a multi-phase system consisting of three phases, i.e. soil particle, gas, water [1,2]. Thermal conductivity of soils is one of the most important parameters because of its important role in environment, earth science, and engineering applications [1]. Results from Refs. [3–11] show thermal conductivity of soils is determined by many factors: mineralogical composition, particle size, gradation, packing geometry, dry density, porosity, water content, cementation, temperature, pore size, pore shape, pore orientation, and spatial arrangement of pores, etc. According to Ref. [12], these influencing factors can be classified into three types: (1) compositional factors, including mineralogical composition, particle size, shape, gradation, interparticle physical contact, etc. (2) environmental factors, including water content, density, temperature, etc. and (3) other factors, including properties of soil components, ions, salts, additives, and hysteresis effect, etc. Two methods, i.e. experimental measurement and model calculation, have been put forward to obtain the thermal conductivity of soils. The experimental measurement uses steady state method and transient state method to measure the thermal conductivity of soils [13,14]. However, it is usually time-consuming, expensive, limited, and only available in certain conditions [1,22]. Thus, a considerable number of researches have been focused on the development of calculation models for the thermal conductivity [15–22].

Dong et al. [2] suggested the calculation models for the thermal conductivity can be divided into the following three categories:

(1) Mixing models (theoretical models), obtained by using the series model and parallel model to connect each component in the cubic cell or representative elementary volume (REV).
(2) Mathematical models, derived from the predictive models of the other properties, such as dielectric permittivity, electrical conductivity, and hydraulic conductivity.
(3) Empirical models, based on the relationship between the effective thermal conductivity and water content or degree of saturation (*S*).

Mixing models are developed by the simplified derivation process with some assumptions, e.g. uniform particle shape, single particle size, series model and parallel model for heat transfer. However, the formula is very complex and it's usually difficult to determine the parameters of the mixing models [12]. Mathematical models are developed by the predictive models of the other properties (e.g. dielectric permittivity, electrical conductivity, and hydraulic conductivity), by assuming the similarity between the thermal conductivity and the other soil properties without consideration of the influencing factors, such as particle size, particle shape, packing geometry, mineralogy composition, temperature, stress level and cementation [3,5,23–27]. Empirical models have been widely

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used to fit the experimental data by various mathematical formulas between the thermal conductivity and water content. They have simple formulas and high prediction accuracy [12,19].

Besides, there exists a class of models that considers the effect of microstructural features on the thermal conductivity of soils. Likos [28] obtained the thermal conductivity at full range of *S* from pore-scale thermal conductivity of a single unit pore by upscaling method. Chen et al. [9–11] proposed a homogenization-based model to calculate the thermal conductivity of unsaturated soils with considerations of the microstructural features of soils, e.g. size, shape, orientation, and spatial arrangement of pores. Compared with empirical models, more factors are considered in the models developed by the upscaling method or the homogenization-based method, leading to the complicate formulas. Therefore, the empirical model was chosen in this study to model the relationship between the thermal conductivity and *S* at full range of *S* (thermal conductivity curve, TCC). Moreover, the new model should be constrained by the Wiener bounds and Hashin-Shtrikman bounds (H—S bounds) [8,29]. The Wiener bounds, representing the lower and upper values of thermal conductivities, are usually used to constrain thermal conductivity models for any mixtures [12,29] while the H—S bounds proposed by Hashin and Shtrikman [30] are usually used to constrain thermal conductivity models for isotropic mixtures. Besides, the H—S bounds are narrower than the Wiener bounds [8,29].

The objective of the research is to present a new model for calculation of the thermal conductivity for fine-grained soils at full range of *S*. Three parameters of the new model were determined by two equations and a measured point at a certain *S*. Experimental data of 36 soils were collected from the Refs. [16,32,33], of which 30 Canadian soils were used to develop the equations while 6 Chinese soils were used to verify the new model.

2. A new model for TCC

Lu and Dong [34] proposed a TCC model based on the similarity between soil water characteristic curve (SWCC) and TCC, and then used the parameters of SWCC model to estimate TCC. He et al. [1] indicated the similar expressions of TCC and SWCC, and proposed a new model for calculation of TCC. Inspired by their work [1,34–36], the Fredlund and Xing model for SWCC (Eq. (1)) [36] was modified to model the relationship between K_e and *S* (Eq. (2)). K_e (Eq. (3)) is defined as the dimensionless thermal conductivity [31], combination of Eqs. (2) and (3) will lead to Eq. (4). Besides, the relationship between K_e and *S* is defined as the dimensionless thermal conductivity curve (dimensionless TCC).

$$\left.\begin{aligned} \theta &= \theta_s C_\psi \frac{1}{\left(\ln\left(\mathrm{e}+\left(\frac{\psi}{a}\right)^c\right)\right)^b} \\ C_\psi &= 1-\frac{\ln(1+\psi/C_r)}{\ln(1+\psi_0/C_r)} \end{aligned}\right\} \tag{1}$$

where C_ψ is the correction factor; C_r is the input value [37]; θ is the volumetric water content; θ_s is the saturated volumetric water content; ψ is the matrix suction; ψ_0 is the highest suction corresponding to zero water content, taken as 10^6 kPa [36]; e is the Euler's number, taken as 2.71828 [38,39]; *a*, *b* and *c* are the curve-fitting parameters.

$$K_e = \begin{cases} 0 & S = 0 \\ \frac{1}{\left(\ln\left(\mathrm{e}+\left(\frac{a}{S}\right)^c\right)\right)^b} & S > 0 \end{cases} \tag{2}$$

where K_e is the dimensionless Kersten number [15]; *S* is the degree of saturation (*S*).

$$K_e = \frac{\lambda - \lambda_{dry}}{\lambda_{sat} - \lambda_{dry}} \tag{3}$$

where λ is the thermal conductivity; λ_{dry} is the thermal conductivity at dried condition; λ_{sat} is the thermal conductivity at saturated condition.

$$\lambda = \begin{cases} \lambda_{dry} & S = 0 \\ \lambda_{dry} + (\lambda_{sat} - \lambda_{dry}) \frac{1}{\left(\ln\left(\mathrm{e}+\left(\frac{a}{S}\right)^c\right)\right)^b} & S > 0 \end{cases} \tag{4}$$

3. Parameters of the new model

3.1. Effects of the parameters on the dimensionless TCC

Effect of the parameter *a* on the dimensionless TCC can be seen in Fig. 1, where *a* increases from 0.1 to 0.5 when *b* = 1.5, *c* = 4. Clearly, the parameter *a* indicates the critical water content. Critical water content is defined to indicate the water content at which the thermal resistivity increases disproportionately with small reduction in water content [40,41]. Zhang et al. [27] also suggested the thermal resistivity will reach a stable region when the water content increases beyond the critical water content. As the thermal conductivity is in inverse proportion to the thermal resistivity [5], the thermal conductivity will also enter into a stable region when the water content increases beyond the critical water content.

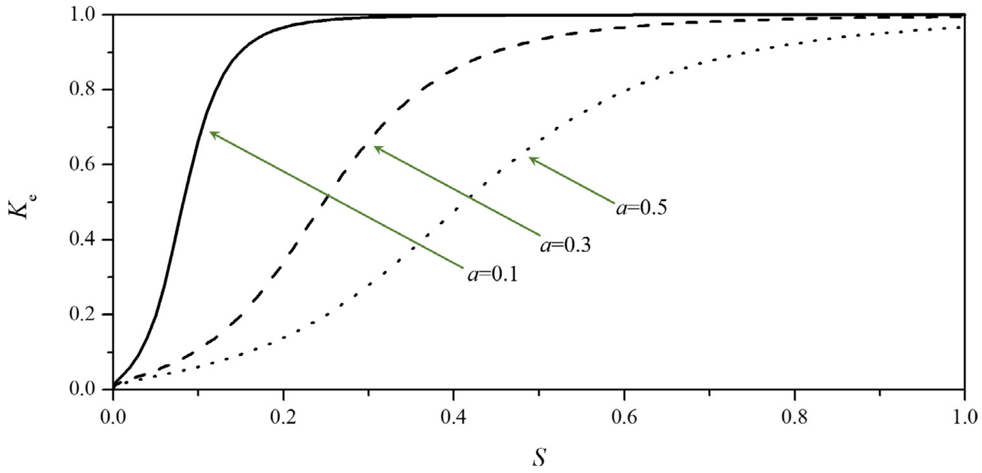

Fig. 1. Relationships between K_e and *S* when varying the parameter *a*, *b* = 1.5, and *c* = 4.