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Laminar mixed convection of power-law fluids in cylindrical enclosures with heated rotating top wall

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ABSTRACT

Laminar mixed convection of inelastic non-Newtonian fluids obeying a power law model in a cylindrical enclosure with a heated rotating top cover has been investigated numerically in this study. The steady-state axisymmetric simulations have been carried out for a range of different nominal Reynolds, Prandtl, Richardson numbers (i.e. $500 \leqslant Re \leqslant 2000; 10 \leqslant Pr \leqslant 500$ and $0 \leqslant Ri \leqslant 1$) and power-law index (i.e. $0.6 \leqslant n \leqslant 1.8$) for an aspect ratio (height/radius) of unity (i.e. $AR = 1.0$). It has been found that mean Nusselt number $\overline{Nu}$ increases as Re and Pr increase, whereas $\overline{Nu}$ decreases with increasing values of Ri for shear-thinning (i.e. $n < 1$), Newtonian (i.e. $n = 1$) and shear-thickening (i.e. $n > 1$) fluids. It has also been observed that the variation of $\overline{Nu}$ with n differs depending on the values of Re and Ri. For instance, for small values of Reynolds number, $\overline{Nu}$ exhibits a non-monotonic trend (i.e. increases before reaching a maximum followed by a decreasing trend) with increasing n for small values of Richardson number, whereas $\overline{Nu}$ monotonically increases with increasing values of n for high Richardson number cases. However, in the case of high Reynolds number, $\overline{Nu}$ increases with n before reaching a maximum value which is followed by a decreasing trend for all values of Ri considered here. Detailed physical explanations are provided for the influences of Re, Pr, Ri, and n on $\overline{Nu}$ based on an elaborate scaling analysis. Finally, the numerical findings have been used to propose a correlation for $\overline{Nu}$ for the ranges of Re, Pr, Ri, n considered here.

1. Introduction

Mixed convection in cylindrical enclosures with a rotating end wall has been extensively investigated [1–14] due to its wide ranging engineering applications such as in fluid machinery, heat exchangers with a rotating fluid, food and chemical processing. Most of the studies in the existing literature are restricted to the Newtonian Fluids (where the viscous stress is directly proportional to strain rate) [1–9]. However, relatively limited effort has been directed to the analysis of mixed convection of non-Newtonian fluids [10–14] (where there is a non-linear relation between viscous stress and strain rate) despite their immense practical importance in anaerobic digesters, bio-chemical synthesis and polymer processing, to name a few notable applications.

Laminar flow of Newtonian fluids in a cylindrical container with a rotating cover has been experimentally investigated by Vogel [1,2], Ronnenberg [3], and Bertela and Gori [4]. The formation of different vortical structure and vortex breakdown phenomenon in this configuration have been reported for different values of Reynolds number (i.e. $\Omega R^2/\nu$ where Ω is the angular speed and ν is the kinematic viscosity) and aspect ratio (i.e. height to radius ratio $AR = H/R$) of the enclosure [1–4]. Escudier [5] extended these findings to provide stability criterion for vortex breakdown. The formation of vortical structures in this configuration has been found to considerably affect the rate of heat transfer in Newtonian fluids [8,9]. Moreover, the effects of Prandtl [8], Reynolds and Richardson numbers [9] on the flow pattern and heat transfer rate in cylindrical enclosures with a heated rotating top wall have also been investigated for an aspect ratio of unity (i.e. $AR = H/R = 1$). It has been found that the mean Nusselt number is a strong function of Prandtl number [8] and the advective transport weakens, while the diffusive transport strengthens with an increase in Richardson number [9].

Several studies also focused on the flow structure of non-Newtonian fluids [10–12] in cylindrical enclosures with a rotating end wall. Escudier and Cullen [10] analysed cylindrical enclosures experimentally with a rotating top cover for viscoelastic shear-thinning fluids (i.e. viscosity decreases with increasing strain rate

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Nomenclature

a	bridging function [–]
AR	aspect ratio ($AR = H/R$) [–]
b	bridging function [–]
C	correlation parameter [–]
c_1, c_2, c_3	correlation parameters [–]
c_p	specific heat at constant pressure [J/kg K]
D	correlation parameter [–]
d_1, d_2, d_3	correlation parameters [–]
e_a	relative error [–]
f_1	functions [–]
g	gravitational acceleration [m/s^2]
Gr	Grashof number [–]
h	heat transfer coefficient [W/m^2 K]
H	height of cylindrical enclosure [m]
k	thermal conductivity [W/mK]
k_0, k_1	correlation parameters [–]
m_0, m_1	correlation parameters [–]
n	power-law index [–]
Nu	Nusselt number [–]
$\overline{Nu}$	mean Nusselt number [–]
Pr	Prandtl number [–]
q	heat flux [W/m^2]
R	radius of cylindrical enclosure [m]
Ra	Rayleigh number [–]
Re	Reynolds number [–]
Ri	Richardson number [–]
T	temperature [K]
U	characteristic velocity scales in radial direction (m/s)
V	characteristic velocity scales in tangential direction (m/s)
y_0, y_1, y_2	correlation parameters [–]
α	thermal diffusivity [m^2/s]
β	coefficient of thermal expansion [1/K]
$\dot{\gamma}$	shear rate [1/s]
δ,δ_{th}	hydrodynamic and thermal boundary layer thickness [m]
θ	non-dimensional temperature ($\theta = (T - T_C)/(T_H - T_C)$) [–]
μ	plastic viscosity [Ns/m^2]
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
τ	shear stress [N/m^2]
Ω	angular velocity [1/s]

Subscripts

C	cold wall
eff	effective value
H	hot wall
max	maximum value
nom	nominal value
r	radial direction
ref	reference value
$wall$	wall value
wf	condition of the fluid in contact with the wall
z	axial direction
ϕ	tangential direction

Special characters

ΔT	difference between hot and cold wall temperature (=($T_H - T_C$)) [K]
$\Delta_{\text{min,cell}}$	minimum cell distance [m]
r	grid expansion ratio [–]

and strain rate is dependent on time for constant shear stress). Stokes and Boger [11] proposed a regime diagram for flow stability based on Reynolds and Elasticity numbers for viscoelastic fluids in cylindrical enclosures with a rotating cover. The influence of shear-thinning character of inelastic non-Newtonian fluids on vortex breakdown (observed by Vogel [1,2] and Escudier [5] for Newtonian fluids) in cylindrical enclosures with a rotating cover was analysed both experimentally and numerically by Böhme et al. [12] where the viscosity was approximated by a power-law in terms of the shear rate. Böhme et al. [12] developed an aspect ratio – Reynolds number ($AR - Re$) diagram, representing the domain of vortex breakdown for shear-thinning fluids.

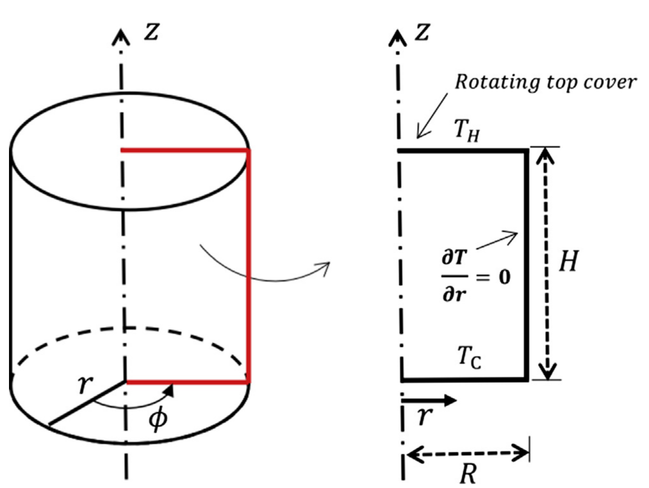

Fig. 1. Schematic diagram of simulation domain and boundary conditions.

Table 1
The details of the meshes and the relative error for the mean Nusselt number $\overline{Nu}$ for shear-thinning (e.g. $n = 0.6$), Newtonian (i.e. $n = 1.0$) and shear-thickening (e.g. $n = 1.8$) fluids for $Ri = 0.5$ and $Re = 2000$ at $Pr = 100$.

Mesh Details	M1 (75 × 75)	M2* (150 × 150)	M3 (300 × 300)
$\Delta_{min,cell}/R$	3.10 × 10^{-3}	1.55 × 10^{-3}	0.78 × 10^{-3}
r_e	1.0519	1.0307	1.0203
Relative Error	M1 (75 × 75)	M2* (150 × 150)	M3 (300 × 300)
$\overline{Nu}$ ($n = 0.6$)	10.673	10.547	10.485
e_a (%)		1.19	0.59
$\overline{Nu}$ ($n = 1$)	23.559	23.314	23.195
e_a (%)		1.05	0.51
$\overline{Nu}$ ($n = 1.8$)	26.338	26.082	25.962
e_a (%)		0.98	0.46

*The mesh which is used for the numerical simulations.