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An element decomposition method for heat transfer analysis

S.Z. Feng [a,*], Y.H. Cheng [b]

[a] School of Mechanical Engineering, Hebei University of Technology, Tianjin 300130, PR China
[b] School of Mechatronic Engineering, China University of Mining and Technology, Xuzhou 221116, PR China

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ABSTRACT

In this work, an element decomposition method (EDM) is formulated to deal with heat transfer analysis. For this method, the quadrilateral elements are first divided into sub-triangular cells and the local temperature gradient in each sub-triangular cell is obtained using linear interpolation function. Then, the temperature gradient of whole quadrilateral can be calculated by averaging the local gradient in each sub-domain. As only one integration point is utilized for each element, the computational cost of presented method is much less and no mapping or coordinate transformation is involved. Several numerical examples are given to fully test the validity of this method and it is found that the EDM is very accurate, stable and can achieve high level of convergence when solving heat transfer problems.

1. Introduction

In practical engineering, structures often work under various thermal loads. In order to ensure the safety of structures, it is quite crucial to obtain the thermal responses of structures through heat transfer analysis [1–3]. For some problems, analytical method can be directly used to get the exact thermal responses of structures. However, for structures with complex geometry and boundary conditions, the analytical solutions are usually very hard to find and hence, numerical methods are widely utilized to deal with such analysis [4–6].

In the past several decades, various numerical methods have been developed and extended to deal with heat transfer problems, such as the finite element method (FEM) [7], the meshfree method [8,9], the finite volume (FV) method [10], the boundary element method (BEM) [11], etc. Among all these methods, the FEM has been proved to be a dominant numerical tool for thermal analysis of engineering structures. The quadrilateral isoparametric element plays an important role in engineering analysis using FEM, which is very accurate and stable. However, it also has some drawbacks. For example, as there are four integration points in each quadrilateral element, the computational cost will be high especially for large scale problems. In addition, the isoparametric coordinate transformation is requisite for the computation and hence, the mesh quality must be guaranteed. If there are severely distorted elements, the determinant of Jacobian matrix will not be positive and the numerical analysis thereupon fails.

Some researches have been done to break through these limitations. To reduce the computation effort when using quadrilateral element, the reduced integration technique has been developed, in which only one integration point is utilized [12]. Although this method can effectively reduce the computational cost, it is not always stable. After that, some studies have been done to improve the stability of one-point integration technique. For example, Belytschko proposed the orthogonal hourglass control method to deal with the perturbation hourglass stabilization and then, this method was successfully extended to solve heat transfer problem [13]. Bachrach introduced the mixed methods for the stabilization of one-point integration, which can provide solutions with high level of accuracy [14]. Unfortunately, all these stabilization methods are limited to mapped quadrilateral elements. Therefore, the reliance on mesh quality still exists and more manual interventions are needed when generating the computational mesh. Besides, some uncertain parameters are used to construct the stabilization matrix for these methods, which are usually hard to determine.

To break through the limitation of mapped elements, Liu et al. proposed the smoothed finite element method (SFEM) by applying the strain smoothing technique to standard FEM [15–18]. In the SFEM, each quadrilateral element is further divided into several smoothing cells and the integrals are computed along the edges of these smoothing cells based on the Green's divergence theorem. Therefore, no mapping and isoparametric coordinate transformation is involved during the analysis. In the SFEM formulation, the shape of quadrilateral element can be arbitrary and numerical analysis can still be implemented even when severely distorted elements exist. The SFEM has been successfully used to solve heat

* Corresponding author.
E-mail address: szfenghnu@126.com (S.Z. Feng).

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transfer problems, which can provide ultra accurate solutions and is much less sensitive to mesh distortions. Although the SFEM possesses some good characteristics, the related computational cost is also increased.

In order to effectively overcome these drawbacks, this study introduces the element decomposition method (EDM) for the heat transfer analysis of structures. In the EDM, each quadrilateral element is directly divided into four sub-triangular cells and only one integration point is used to construct the elemental conduction matrix. No mapping technique or coordinate transformation process is involved in the numerical implementation. Numerical examples with various kinds of boundary conditions are presented to fully investigate the validity of the EDM for heat transfer analysis through comparing results with those obtained by traditional four-point integration quadrilateral element.

2. Governing equations for heat transfer analysis

It is assumed that the material obeys Fourier's Law of heat conduction and the differential equation governing heat conduction can be given as

$$\left(k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2}\right) + \rho Q = 0 \tag{1}$$

where T denotes the temperature, Q represents the internal heat generation, ρ is the density, k_x and k_y are the thermal conductivities in the x, y directions, respectively. The thermal boundary conditions are directly given as

$$\text{Dirichlet boundary}: \ T|_\Gamma = T_w \tag{2}$$

$$\text{Neumann boundary}: \ -k\frac{\partial T}{\partial n_0}|_\Gamma = q \tag{3}$$

$$\text{Robin boundary}: \ -k\frac{\partial T}{\partial n_0}|_\Gamma = h_c(T - T_\infty) \tag{4}$$

$$\text{Adiabatic boundary}: \ -k\frac{\partial T}{\partial n}|_\Gamma = 0 \tag{5}$$

where T_w represents the known temperature, q denotes the prescribed heat flux, h_c is the convection coefficient, T_∞ represents the environmental temperature, n_0 is the unit outward normal to the boundary and Γ represents the boundary.

3. EDM for thermal analysis

3.1. EDM formulation

This section presents the EDM formulation for thermal analysis of structures. At first, the problem domain is discretized using quadrilateral elements. By connecting the central point of quadrilateral element to the field nodes, each quadrilateral element is further divided into four sub-triangular cells Δ_1, Δ_2, Δ_3 and Δ_4, as shown in Fig. 1. It is clearly seen that $\Omega_e = \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \Delta_4$, $\Omega_i \cap \Omega_j = \emptyset \ (i \neq j, \ i,j = 1, 2, 3, 4)$. The coordinates of central point can be simply given as

$$x_c = (x_1 + x_2 + x_3 + x_4)/4 \tag{6}$$

$$y_c = (y_1 + y_2 + y_3 + y_4)/4 \tag{7}$$

Then, the temperature of central point is given as the average of four field nodes.

$$T_c = (T_1 + T_2 + T_3 + T_4)/4 \tag{8}$$

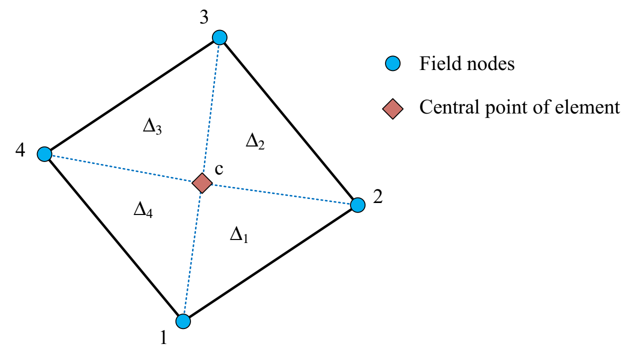

Fig. 1. Division of a quadrilateral element into four sub-triangles.

It should be noted that this is an approximation of the temperature of central point and it may be not accurate enough when the element is extremely distorted. For each sub-triangular cell of the quadrilateral element, the temperature field is approximated using linear interpolation functions. For convenience sake, we just give the formulations in sub-triangular cell Δ_1 and the deducing processes in the other sub-triangular cells are quite similar.

In sub-triangular cell Δ_1, the temperature field is interpolated by

$$T_{\Delta_1} = N_1 T_1 + N_2 T_2 + N_3 T_c \tag{9}$$

where N_i is the linear shape function, which can be given as

$$N_i = a_i + b_i x + c_i y \tag{10}$$

$$a_i = \frac{1}{2\mathrm{A}_{\Delta_1}}(x_j y_k - x_k y_j), \ b_i = \frac{1}{2\mathrm{A}_{\Delta_1}}(y_j - y_k), \ c_i = \frac{1}{2\mathrm{A}_{\Delta_1}}(x_k - x_j) \tag{11}$$

where A_{Δ_1} denotes the area of sub-triangular cell Δ_1. The temperature gradient can be expressed as

$$\mathbf{g}^{\mathrm{T}} = \begin{bmatrix} \frac{d\mathbf{T}}{dx} & \frac{d\mathbf{T}}{dy} \end{bmatrix} \tag{12}$$

The local temperature gradient in sub-triangular cell Δ_1 can be calculated by

$$\mathbf{g}^{\Delta_1} = \begin{bmatrix} B_1^{\Delta_1} & B_2^{\Delta_1} & B_3^{\Delta_1} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_c \end{bmatrix} \tag{13}$$

where,

$$B_1^{\Delta_1} = \begin{bmatrix} b_1 \\ c_1 \end{bmatrix}, \ B_2^{\Delta_1} = \begin{bmatrix} b_2 \\ c_2 \end{bmatrix}, \ B_3^{\Delta_1} = \begin{bmatrix} b_3 \\ c_3 \end{bmatrix} \tag{14}$$

Substituting Eqs. (8) and (9) into Eq. (13), the temperature gradient of sub-triangular cell Δ_1 can be rewritten as

$$\mathbf{g}^{\Delta_1} = \begin{bmatrix} B_1^{\Delta_1} + \frac{1}{4}B_3^{\Delta_1} & B_2^{\Delta_1} + \frac{1}{4}B_3^{\Delta_1} & \frac{1}{4}B_3^{\Delta_1} & \frac{1}{4}B_3^{\Delta_1} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \tag{15}$$

Then, the temperature gradient of the other sub-triangular cells can be obtained in the same way.

$$\mathbf{g}^{\Delta_2} = \begin{bmatrix} \frac{1}{4}B_3^{\Delta_2} & B_1^{\Delta_2} + \frac{1}{4}B_3^{\Delta_2} & B_2^{\Delta_2} + \frac{1}{4}B_3^{\Delta_2} & \frac{1}{4}B_3^{\Delta_2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \tag{16}$$