



Heat and mass transfer in a rarefied gas confined between its two parallel condensed phases

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ABSTRACT

The heat and mass transfer in a rarefied gas between its two parallel condensed phases is considered on the basis of linearized and non-linear S-model kinetic equations. The profiles of the macroscopic parameters in the gap between gas-liquid interfaces are obtained for several Knudsen numbers and for the cases of complete and non-complete evaporation and condensation. The linearized Navier–Stokes equations and energy equation, subjected to the temperature and pressure jump boundary conditions, are solved analytically and the expressions for the macroscopic parameters are obtained. The comparison of three approaches allowed us to establish the limit of the application of the linearized approaches in term of the saturation temperature ratio between the condensed phases.

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1. Introduction

Evaporation and condensation phenomena are interesting not only from the fundamental point of view, but they are also largely present in various industrial applications. During the evaporation and condensation the liquid-vapor interface is in non-equilibrium state and its detailed modeling requires the use of the molecular based approaches, like the kinetic theory of gases [1] or the molecular dynamics [2]. The adequate description of the evaporation-condensation was in focus of interest from a long time [3–8,1]. However, the complete understanding of these phenomena is just far to be completely achieved. In addition, the implementation of the kinetic models or the molecular dynamics based approaches is not so easy for the practical problems. This is why it is also interesting to use the continuum approaches, like the classical Navier–Stokes equations, subjected to the temperature and pressure jumps boundary conditions [9,10] to simulate the evaporation-condensation problems. Recently the alternative macroscopic approach based on the regularized 13 moments equations [11] was also proposed and applied for the simulation of the evaporation and condensation phenomena.

The main objective of this study is the development of the numerical approach to simulate the behavior of the vapor phase between two parallel liquid-vapor interfaces. Contrarily to the authors of Ref. [8], where the BGK equation is implemented, the

linearized and non-linear S-model [12] kinetic equations are solved numerically to simulate the evaporation-condensation phenomena appearing between these two plane condensed phases. The main advantage of the S-model is its capacity to provide the correct Prandtl number, equals to 2/3, for the monatomic gases, which can be important, when the problems involving the vapor flows and the heat flux are considered. In addition, the linearized Navier–Stokes equations and energy equation subjected to the temperature and pressure jumps boundary conditions [9,10] are also solved analytically and the simple relations are obtained for the profiles of the macroscopic parameters of vapor in the gap between two condensed phases. The influence of evaporation-condensation coefficient on the macroscopic parameters is also analyzed. Comparing the results obtained by three approaches we establish the limit of the applicability of the linearized kinetic equation as well as of the linearized Navier–Stokes equations for the simulation of the evaporation and condensation phenomena.

2. Problem statement

We consider two parallel plane condensed phases at rest, maintained at temperature T_1 and T_2 , on the bottom ($y' = 0$) and top ($y' = H$) interfaces, respectively, y' is the coordinate normal to both condensed phases. Let p_1 and p_2 be the saturation gas pressures at temperatures T_1 and T_2 , ($T_1 > T_2$), respectively. We investigate here the behavior of the monatomic gas motion caused by evaporation and condensation on the condensed phases, first on the basis of the kinetic theory.

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If we consider that the ratios p_1/p_2 and T_1/T_2 are independent, then the evaporation–condensation problem is completely characterized by three parameters: p_1/p_2 and T_1/T_2 ratios and by the rarefaction parameter δ . However, in practice, the saturation pressure and temperature are related by the Clausius–Clapeyron equation [13] and therefore cannot take the arbitrary values. In this case the evaporation–condensation problem depends on two parameters only: the slop of $p'(T')$ function and the rarefaction parameter δ . In the following we consider both cases: first the situation when two parameters are independent, and then, the cases of Argon and several other monatomic gases, where the saturation pressure and temperature are related through the Clausius–Clapeyron equation.

The distance H between two interfaces is taken as the characteristic dimension of the problem, so the rarefaction parameter δ is defined as following:

$$\delta = \frac{H}{\ell}, \quad (1)$$

where ℓ is the equivalent mean free path:

$$\ell = \frac{\mu_2 v_2}{p_2}, \quad (2)$$

which is defined using the reference parameters with subscript 2, corresponding to the upper plate. The choice of the parameters related to the upper plate as the reference parameters was done without loss of generality. In Eq. (2) $\mu_2 = \mu(T_2)$ is the dynamic viscosity of the vapor phase

$$\mu(T') = \mu_2 \sqrt{\frac{T'}{T_2}}, \quad (3)$$

v_2 is the most probable molecular speed,

$$v(T') = \sqrt{2\mathcal{R}T'}, \quad (4)$$

calculated at the temperature T_2 : $v_2 = v(T_2)$, $\mathcal{R} = k_B/m$ is the specific gas constant, k_B is the Boltzmann constant, m is the molecular mass.

3. Modeling based on the kinetic equation

To model the evaporation and condensation phenomena between two parallel condensed phases maintained at different temperatures first the S-model kinetic equation [12] is used. The evaporation–condensation problem between two parallel condensed phases is considered here as steady state and one dimensional in physical space, so the S-model kinetic equation is written as

$$v_y \frac{\partial f}{\partial y'} = v(f^S - f), \quad (5)$$

where $f(y', \mathbf{v})$ is the one particle velocity distribution function, $\mathbf{v} = (v_x, v_y, v_z)$ is the molecular velocity vector, v is the collision frequency, $v = p'/\mu$. In the frame of S-model the equilibrium distribution function f^S in Eq. (5) is defined as following

$$f^S(y', \mathbf{v}) = f^M \left[1 + \frac{2m\mathbf{V}\mathbf{q}'}{15n'(y')(k_B T'(y'))^2} \left(\frac{m\mathbf{V}^2}{2k_B T'(y')} - \frac{5}{2} \right) \right], \quad (6)$$

here $T'(y')$ is a gas temperature, $n'(y')$ is a gas number density, $\mathbf{u}' = (0, u', 0)$ is a bulk velocity vector, $\mathbf{V} = \mathbf{v} - \mathbf{u}'$ is the peculiar velocity vector, $\mathbf{q}' = (0, q', 0)$ is a heat flux vector, f^M is the Maxwellian distribution function [14]. The macroscopic parameters are defined as follows:

$$n'(y') = \int f(y', \mathbf{v}) d\mathbf{v}, \quad u'(y') = \frac{1}{n'} \int f(y', \mathbf{v}) v_y d\mathbf{v}, \quad (7)$$

$$T'(y') = \frac{m}{3k_B n'} \int f(y', \mathbf{v}) V^2 d\mathbf{v}, \quad q'(y') = \frac{m}{2} \int f(y', \mathbf{v}) V^2 (v_y - u'_y) d\mathbf{v}. \quad (8)$$

By integrating Eq. (5) multiplied by $(1, v_y, v_y^2)$ we obtain the integrals expressing the number of molecules, y component of momentum and energy transported in y direction per a unit area of a plan, parallel to the condensed phases. The evaporation flow rate, expressed in the number of molecules per time and per surface unit, J'_n , and the evaporation mass flow rate, expressed in kilogram per time and surface units, J'_ρ , are defined as followings:

$$J'_n = \int v_y f(y', \mathbf{v}) d\mathbf{v}, \quad J'_\rho = m \int v_y f(y', \mathbf{v}) d\mathbf{v}. \quad (9)$$

The second definition of the evaporation mass flow rate is usually provided from the experiments. The energy flux is defined as

$$J'_E = \int v_y v^2 f(y', \mathbf{v}) d\mathbf{v}. \quad (10)$$

The constancy of the integrals J'_n and J'_E will be further used to estimate the accuracy of the numerical calculations and for the comparison with the results of Ref. [8].

3.1. Linearized S-model equation

To linearize S-model kinetic Eq. (5) we assume that deviations between the temperatures of two condensed surfaces and the corresponding saturation pressures are small:

$$X_p = \frac{p_1 - p_2}{p_2} \ll 1, \quad X_T = \frac{T_1 - T_2}{T_2} \ll 1. \quad (11)$$

In previous expressions X_p and X_T can be related to the thermodynamic forces [15]. As it was mentioned in Section 2, for a given gas the pressure and temperature differences are coupled by the relation

$$p_1 - p_2 = \beta(T_1 - T_2), \quad (12)$$

where β is a positive constant corresponding to the slop of the Clausius–Clapeyron curve at T_2 , so X_p and X_T are not independent quantities. However here we consider two forces separately, to see clearly the impact of each force to the evaporation–condensation process.

For further derivation we introduce the following dimensionless quantities:

$$y = \frac{y'}{H}, \quad \mathbf{c} = \frac{\mathbf{v}}{v_2}, \quad \mathbf{u} = \frac{\mathbf{u}'}{v_2}, \quad n = \frac{n'}{n_2}, \quad T = \frac{T'}{T_2}, \quad p = \frac{p'}{p_2}, \quad \mathbf{q} = \frac{\mathbf{q}'}{p_2 v_2}. \quad (13)$$

When X_p and X_T are small enough compared to 1 we can linearize the distribution function as following:

$$f = f_0^M (1 + h_p X_p + h_T X_T), \quad (14)$$

where h_p and h_T are the perturbation functions, related to the pressure and temperature difference, respectively, f_0^M is the absolute Maxwellian distribution function with the reference number density, n_2 and temperature, T_2 :

$$f_0^M = n_2 \left(\frac{m}{2\pi k_B T_2} \right)^{3/2} \exp(-c^2). \quad (15)$$

For the real gases, for any value of β we can obtain the solution as a superposition of two functions as

$$h = (\beta h_p + h_T) \Delta T, \quad (16)$$

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