



A generalized Characteristic-Based Split projection method for Navier-Stokes with real fluids

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ABSTRACT

A new computational approach for the general numerical simulation of fluid flows is presented and demonstrated for compressible natural convection flows with real fluid properties. The Characteristic-Based Split finite element method is generalized, and abbreviated as GCBS, in the present work to introduce both pressure-expansion effects in addition to thermal-expansion effects on density within the projection step, in contrast to recent works that preserve only one of those two expansion effects. Moreover, we keep the fluid in thermodynamic equilibrium to support a real equation of state and fluid model. The method retains the full Navier-Stokes equations, transitioning smoothly between compressible and incompressible flows, without special artificial-compressibility methods, special low-Mach-number methods, or other modeling treatments. Neither isothermal or isobaric assumptions are made. The non-linear solution obtained is validated by the linearized solution during the early time period of the simulation. Natural convection in a cavity is explored for its convenience in testing across known benchmark flow regimes, from classical Boussinesq flow to the full Navier-Stokes equations with real fluid models. Results for natural convection in a cavity across Rayleigh numbers, non-dimensional adiabatic temperature gradients, and temperature differences, are obtained across flow regimes with the GCBS method. Classical Boussinesq, Thermodynamic Boussinesq, low-Mach number, and real-gas flows are presented, demonstrating the generalization of the method across flow regimes. Thus, our contribution is to generalize the Characteristic-Based Split method to capture the full Navier-Stokes equations while demonstrating the capability with compressible natural convection.

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1. Introduction

In this paper we further develop the Characteristic-Based Split (CBS) method [1,2] into a generalized CBS method, abbreviated as GCBS, for solving the full compressible Navier-Stokes equations by including thermal-expansion effects, which we demonstrate by resolving natural convection without requiring Boussinesq approximations. The CBS method, introduced in 1995, has successfully demonstrated its generality by solving both compressible and incompressible flows [1–3], shallow-water equations [4], and incompressible flow problems with heat transfer through the Boussinesq approximation [5,6]. A major difficulty arises in directly solving non-Boussinesq flows in conservative form with the original CBS method because its projection does not include isobaric compressibility. Recent modifications to the SIMPLE

algorithm [7] were made to include isobaric compressibility, but not isothermal compressibility. Here the general equation of state with thermal and bulk expansion (isothermal compressibility) is used, bringing to the CBS method support for compressible natural convection with a general equation of state and non-constant thermal and transport properties. The resulting general semi-implicit formulation for compressible non-Boussinesq flows is implemented and validated against previous works when Boussinesq [8,9], Thermodynamic Boussinesq [10] and non-Boussinesq ideal-gas assumptions are imposed before exploring real-gas flows where no simplifications of the Navier-Stokes equations or equations of state are made.

The objective of the present work is to generalize the CBS method to extend it to the compressible non-Boussinesq flow regime with real-fluid properties and with conservation laws in conservative form, producing the first projection method capturing the full compressible Navier-Stokes equations. We require that the new formulation be validated with existing benchmark solutions for Boussinesq flows [11,12,6,13–15], Thermodynamic Boussinesq

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flows [10], and compares well with low-Mach number non-Boussinesq flows [16]. It is also required that we maintain all of the terms of the Navier-Stokes equations, particularly in the energy equation to include pressure work, viscous heating, and body-work terms. We further require the method to be capable of using real-fluid equations of state and properties through any general external library, such as RefProp [17], as the static pressure, temperature, and velocity distributions are expected to be very sensitive to them [18]. Mass is required to be conserved within the domain without special treatment, such as the pressure/temperature scaling used in low-Mach number methods [19]. Thus the advantage of the proposed method is a unified approach for solving fully-compressible to fully-incompressible thermal convection problems with arbitrarily large temperature variations.

The CBS method has its origin in an operator-splitting (or projection) method, first introduced by Chorin [20], where velocity and pressure are directly integrated for incompressible flows. Kim and Moin [21,22] then improved Chorin's split with a fractional-step method accurate for transient incompressible flows. The CBS method then extended operator splitting capabilities further to compressible and incompressible flows, and is a topic of continued active research [23,24]. Here we generalize the CBS method by including the thermal expansion term in the equation of state. The split equations are then cast into weak form, which is then discretized by a finite element method. Real-fluid properties from RefProp are introduced efficiently into the code through a new unstructured self-tuning two dimensional interpolation library. Convergence is accelerated with an adaptive mesh refinement scheme. To improve numerical behavior, the pressure field and the energy field are each additively decomposed into a static component and a dynamic component. The conservation-of-energy equation is given an implicit treatment to maintain the CBS Courant-Friedrichs-Lewy (CFL) limit. Artificial compressibility was not required or used for the strictly incompressible-flow limit, as customarily employed when approaching the incompressibility limit [25,26]. The formulation is independent of the element type; however, Lagrange elements used for all scalar fields are of higher order than the Lagrange elements used for all vector fields.

2. Problem statement

Natural convection with the Usual-Boussinesq (UB), Thermodynamic-Boussinesq (TB), ideal-gas, and real-fluid models, with a general equation of state, are considered over the time interval $[0, t_{end}]$, in a domain $\Omega \in \mathbb{R}^d$, within the boundary Γ . The domain, shown in Fig. 1, has a characteristic length, L^* , equal to the edge length, where the asterisk superscript $*$ is used to indicate dimensional values. The characteristic velocity is chosen to be $V^* = \alpha_{ref}^*/L^*$, where α_{ref}^* is the thermal diffusivity of the fluid at the reference state. The reference condition for thermodynamic and transport properties are fixed at a reference pressure, P_{ref} , and a reference temperature, T_{ref} . The initial conditions for the cavity are that of the fluid at rest at the reference temperature and pressure. The fluid motion is governed by the Navier-Stokes equations, of which the continuity, momentum, and energy conservation equations are written in the following non-dimensional form:

Mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial U_i}{\partial x_i} = 0 \quad (1)$$

Momentum conservation

$$\frac{\partial U_i}{\partial t} = -\frac{\partial}{\partial x_j} (u_j U_i) - \frac{1}{Re} \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial \mu \tau_{ij}}{\partial x_j} + Ri \rho f_i \quad (2)$$

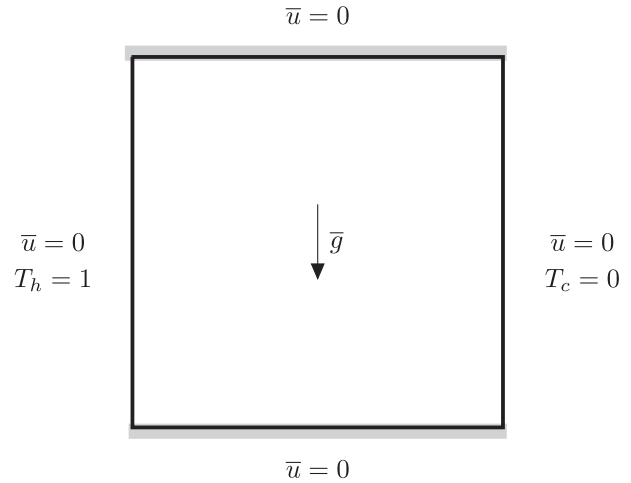


Fig. 1. Natural convection in a cavity heated on the left wall, cooled on the right wall, with the remaining walls adiabatic.

Energy conservation

$$\begin{aligned} \frac{\partial \rho E}{\partial t} = & -\frac{\partial}{\partial x_j} (U_j E) + \frac{1}{RePr} \frac{\partial}{\partial x_j} \left(k \frac{\partial T}{\partial x_j} \right) - \frac{Ec}{Re} \frac{\partial}{\partial x_j} (u_j p) \\ & + \frac{Ec}{Re} \frac{\partial}{\partial x_j} (\mu \tau_{ij} u_i) + Ec Ri f_i U_i, \end{aligned} \quad (3)$$

where ρ is the mass density, u_i the i th component of the velocity vector field (with $i = 1, 2$, or 3 depending on the dimension of the problem), $U_i = \rho u_i$ the i th component of the momentum vector field, p the pressure, f_i the i th component of the body acceleration vector field, E the total energy, T the temperature, k the thermal conductivity, and μ the dynamic viscosity. All of these quantities are non-dimensional. A non-dimensionalization of the equations introduces the Reynolds (Re), Richardson (Ri), Prandtl (Pr), and Eckert (Ec) numbers,

$$Re = \frac{V^* L^* \rho_{ref}^*}{\mu_{ref}^*}, \quad Ri = \frac{g_{ref}^* L^*}{V^{*2}}, \quad Pr = \frac{\mu_{ref}^* c_{p,ref}^*}{k_{ref}^*}, \quad Ec = \frac{V^{*2}}{c_{p,ref}^* \Delta T_{ref}^*},$$

where dimensional values are denoted with the "asterisk" superscript $*$. Here, ρ_{ref}^* is a reference density, μ_{ref}^* a reference dynamic viscosity, g_{ref}^* a reference body-acceleration, $c_{p,ref}^*$ a reference specific-heat, k_{ref}^* a reference thermal-conductivity, and ΔT_{ref}^* a reference temperature-difference set equal to the applied temperature-difference of $T_h - T_c$. The non-dimensional deviatoric stress tensor is given by

$$\tau_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij}. \quad (4)$$

The additional non-dimensional number, the Rayleigh number, is

$$Ra = \frac{\beta_{ref}^* \Delta T_{ref}^* g_{ref}^* L^{*3}}{V_{ref}^* \alpha_{ref}^*} = \frac{Ri}{Pr} \beta_{ref}^* \Delta T_{ref}^*, \quad (5)$$

where in the case of an ideal gas, $\beta_{ref}^* \Delta T_{ref}^*$ becomes $\Delta T_{ref}^*/T_{ref}^*$.

A relevant Mach number is found by scaling against the buoyant velocity scale, $\tilde{V}^{*2} = \beta_{ref}^* \Delta T_{ref}^* g_{ref}^* L^*$, which keeps the velocity field near unity, independent of the Rayleigh number, the Mach number, and the Eckert number, and is found valuable in the consideration of compressible natural convection. The buoyant Mach number $\tilde{M}$ is defined for a general fluid, and $\tilde{M}_{IG}$ the Mach number for an ideal gas,

$$\tilde{M}^2 = \left(\frac{\tilde{V}^*}{c_{ref}^*} \right)^2 = \frac{Ra}{\gamma \frac{\partial p}{\partial \rho}|_T}, \quad \tilde{M}_{IG}^2 = \frac{Ra Pr Ec}{\gamma - 1} \left(\frac{\Delta T^*}{T^*} \right)_{ref} \quad (6)$$

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