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Exact solution of thermal response in a three-dimensional living bio-tissue subjected to a scanning laser beam

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A B S T R A C T

At present, laser plays an important role in biomedical treatments and surgical techniques. To guarantee patients' safety, a deeply understanding of thermal responses of biological tissues during laser-tissue interaction is required. The existing analytical researches usually explore the bio-thermal process based on a one-dimensional heat conduction model because of the complication of the governing equations in three-dimensional case. The present study developed an analytical solution for the laser-tissue thermal interaction with a moving heat source based on a three-dimensional DPL thermal transfer model. The effects of the capillary vessel system and metabolism were under consideration. The expression of temperature in the cuboid bio-tissue was derived analytically by utilizing the Green's function approach and its accuracy was verified through agreement with a numerical simulation. The influences of the laser moving speed, the spot size of laser beam and the two parameters of relaxation times were discussed.

1. Introduction

The thermal treatment method has been widely used in modern medicine, such as laser tissue soldering [1], hyperthermia [2], infrared irradiation [3], laser surgery [4] and other remedy methods. Laser tissue soldering is a promising technique for tissue fusion but is limited by the lack of reproducibility. To address this issue, Schoni et al. [5] established a highly reproducible and strong tissue fusion device by using ICG packed nanoshells. Huang et al. [6] succeeded in treating the ruptured intestinal tissue by laser irradiation with plasmonic nanocomposites employed. It was suggested that laser welding using the nanocomposites can significantly enhance the tensile strength, leakage pressure and bursting pressure of ruptured intestinal tissue. Koch et al. [7] investigated the uptake of silica and gold nanoparticles engineered for laser-tissue soldering in the brain, in which the risk of thrombosis should be minimized. Bogni et al. [8] developed a finite element model to simulate the temperature distribution within blood vessels during laser tissue soldering. Ting et al. [9] investigated the thermal accumulation induced by the multi-point laser sources during the medical laser therapy. The bio-heat transfer finite element method was employed and the criteria for the safe distance among these laser spots were presented. Tuncer et al. [10] studied the effects of the collateral thermal damage on histopathological diagnosis, which were caused by conventional surgery and laser therapy. And it was confirmed that the carbon dioxide laser was an effective instrument for soft tissue excisional biopsies with minimal intraoperative and postoperative complications and good pain control. Su et al. [11] tested the proper conditions for the laser surgery and predicted the residual thermal damage of surface tissue in medical cosmetology.

Tumor hyperthermia is an efficient method of cancer treatment wherein cancer cells are killed by exposing the body tissue to high-power laser beam. It is required to limit the thermal damage in the tumor cells and to protect the healthy tissue from heating injury. For this purpose, Tang et al. [12] evaluated the effect of near-infrared laser tumor thermotherapy at the molecular, cellular, and physical levels. Singh et al. [13] studied the laser-induced hyperthermia treatment of tumor in a two-dimensional axisymmetric tissue embedded with moderate size blood vessels. Effects of power density, laser exposure time, beam radius, diameter of blood vessel, and volume fractions of nanoshells on temperature spread in the tissue were analyzed. Sazgarnia et al. [14] simulated the heating process of laser photothermal therapy for tumor, which was assisted with gold/gold sulfide nanoshells. Lin et al. [15] employed the differential transformation method to propose a numerical scheme for the temperature distributions in hyperthermia treatment. The effects of variations such as the relaxation time, perfusion rate of blood and thermal conductivity were investigated.

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In the above laser surgical operations, it is significant to control of the laser power and the thermal damage region. The heat transfer mechanism in the target tissue should be ascertained so that the thermal response of human tissue can be derived. The most common theory to describe heat conduction process is the Fourier's law, which has been confirmed as an efficient thermal transfer model in usual cases. Pennes [16] developed a bio-heat transfer model for living tissues on the basis of Fourier's Law. The Pennes model has been successfully utilized in a batch of researches. For example, Yue et al. [17] developed a one-dimensional steady-state bioheat transfer model of living tissues in cylindrical coordinates. Analytical solution was derived with the usage of Bessel's equation. Cui et al. [18] developed a one-dimensional analytical thermal model for the flexible electronic devices integrated with human skin under a constant and pulse power.

The Fourier's law assumes that the heat wave propagates at an infinite speed, which can't fully model the transient problems [19]. To resolve this problem, some modifications have been made on Fourier's law by considering the so-called non-Fourier effect. For instance, Cattaneo [20] and Vernotte [21] developed a hyperbolic heat conduction model, hereinafter referred to as C-V model, by introducing one relaxation time, which allowed the temperature gradient to precede the heat flux vector. Later on, many researchers utilized the C-V model to investigate the heat transfer problems. For example, Mishra et al. [22,23] analyzed the combined mode conduction and radiation heat transfer in two dimensional square and concentric spherical enclosures with non-Fourier effect considered. Ma et al. [24] studied the thermal response of a square plate which is irradiated by a moving temporally non-Gaussian laser pulse.

In addition, Tzou [25] presented another heat conduction model called dual-phase-lag model (DPL model), which allowed either the temperature gradient to precede heat flux vector or the heat flux vector to precede temperature gradient. And this model was widely used in bio-heat process. For example, Fan and Wang [26] developed a general bioheat transfer model at macroscale for biological tissues with the required closure provided. Zhang et al. [27] explored the thermal transfer process in biological tissues with the thermal lag focused on. Finite difference method and DPL model was used to obtain the thermal behavior of the substrate. Afrin et al. [28] studied the nonequilibrium heat transfer in living biological tissues with arterial and venous bloods considered. It was found that the phase lag times for heat flux and temperature gradient only depend on properties of artery, vein and tissue, blood perfusion rate and convective heat transfer rate. Villiger et al. [29] numerically simulated the temperature response of a three-dimensional bio-heat model which was subjected to a moving laser beam. Finite element method was employed and the influence of wavelength on the injury depth was discussed. Lin and Li [30] derived analytical solutions of non-Fourier bio-heat conductions for skin subjected to pulsed laser heating. They modeled skin as a one-dimensional structure.

To solve partial differential equations of heat transfer problems, the eigenfunction expansion technique is widely used. The other commonly used technique is integration transform method. For example, Torabi and Zhang [31] derived the exact analytical solution for DPL model of heat conduction in a cylindrical geometry by using the separation of variables analytical method together with Duhamel's theorem. Kumar and Srivastava [32] developed an analytical solution to the two-dimensional DPL bio-heat transfer equation by using finite integral transform method. Ramadan [33] presented a Laplace transform-based solution procedure for transient heat transfer in a multilayered planar slab, multilayered solid cylinders, and spheres within the framework of the dual phase lag model. Sun et al. [34] investigated the thermoelastic behavior of thin metal film by using Laplace transform approach when it is heated by pulsed laser. Askarizadeh and Ahmadikia [35] presented an exact analytical analysis of two-dimensional Fourier and non-Fourier bioheat transfer equations in skin tissue exposed to an instantaneous heating condition through Laplace transform technique in conjunction with the separation of variables method and inversion theorem. Sun et al. [36] investigated the heat conduction behavior of a two-dimensional axisymmetric laminated structure during pulsed laser heating by utilizing the Green's function approach. In 1985, Frankel et al. [37] presented the Green's function approach for hyperbolic heat conduction in a one-dimensional medium. Ma et al. [38] extended this procedure into DPL model in a two-dimensional finite medium. Yet there is no report about the usage of Green's function approach in 3D non-Fourier heat conduction equations.

Both the non-Fourier effect and moving heating sources appear commonly in practical laser surgery. Yet few literatures treat with the thermal response of substrate subjected to a moving heat source by using the DPL bio-tissue model, because it is complicated to obtain the thermal process in such a case. It is challengeable to treat with the combination of the DPL model and the moving heat source at the same time. To address this issue, the thermal behavior of a three-dimensional living bio-tissue with blood vessel capillaries, which was subjected to a moving source in a laser surgery, was investigated in the present study. The mode superposition method was employed to derive the analytical expression of temperature. The effects of spot size, relaxation times and source scanning speed on the thermal responses were discussed.

2. Mathematical model

The bio-tissue is modeled as a cuboid, as shown in Fig. 1. The length, width and height of the cuboid are L, b and h, respectively. A laser beam with square cross section moves along the central axis of the top surface in x-direction at a uniform speed v.

The bioheat conduction equation may be written as following [39]:

$$\rho C \frac{\partial T}{\partial t} = -\nabla \cdot q + \dot{S} \tag{1}$$

where ρ and C are the density and specific heat of skin tissue, respectively. q is the heat flux vector, T is the temperature, and $\dot{S}$ represents the volumetric source term, which is expressed as

$$\dot{S} = w_b \rho_b C_b (T_a - T) + Q_m + Q_l \tag{2}$$

where ρ_b,C_b and w_b are the density, specific heat and perfusion rate of blood, respectively. T_a is the temperature of arterial blood, Q_m the metabolic heat generation and Q_l the volumetric heat source of the laser beam.

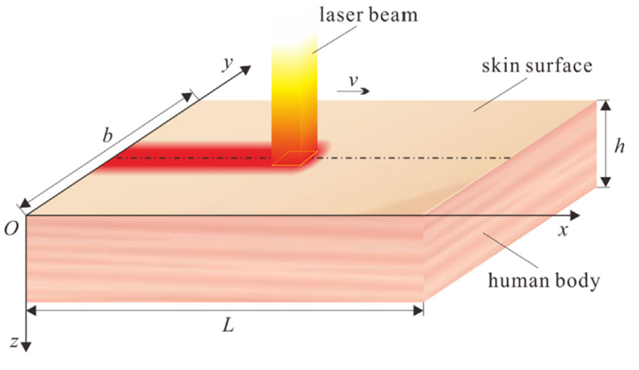

Fig. 1. Illustration of the 3D bio-tissue model.