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The effect of thermocapillarity on the dynamics of an exterior coating film flow down a fibre subject to an axial temperature gradient

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ABSTRACT

The dynamics of a viscous film flowing down a vertical fibre under the action of gravity and the thermocapillarity induced by an axial temperature gradient is investigated theoretically. The instability of this exterior coating flow is driven by a Rayleigh-Plateau mechanism modified by the presence of gravity as well as the thermocapillarity. We derived an evolution equation for the interface in the framework of the long wave approximation. A linear stability analysis and a nonlinear simulation are performed to investigate the influence of the thermocapillarity on the dynamics of axisymmetric disturbances. The results of linear stability showed that the thermocapillarity does not influence the growth rate of the disturbance and only affects its frequency. For the nonlinear evolution, the thermocapillarity plays an important role in influencing the profile of the interface in different flow regimes. We also examined the effect of thermocapillarity on the wave speed and the characteristics of the structures of travelling wave solutions.

1. Introduction

The dynamics of a film flowing down a cylindrical fibre driven by gravity has been much studied due to its technological importance, mainly in the processes of draining, coating of insulation on a wire, and the protection coating of tube walls [1]. Lord Rayleigh [2] was the first to identify the so called Rayleigh-Plateau mechanism by which droplets form under the action of surface tension in cylindrical fluid threads and jets. For an axisymmetric film flow coating a cylindric fibre, the Rayleigh-Plateau instability is modified by the presence of flow driven by gravity.

An isothermal vertical flow down a vertical fibre has been extensively studied. Experimental investigation on the coating flows on fibres driven by gravity was performed first by Quéré [3]. The results showed that the presence of a mean flow modulates the surface-tension-driven flow. For a thick film on a slender fibre, drops develop due to the Rayleigh instability and flow downward. Some of drops grow by swallowing the other ones, and quickly fall, leaving behind them a thick film which breaks in turn into droplets. For a thin film on a large fibre, the break-up process may be arrested by the mean flow.

In the present publications of theoretical works, several modeling approaches have been developed to investigate the dynamics of exterior coating flows on fibres. These models can be loosely categorized into three groups: (i) thin film asymptotic models, (ii) long-wave asymptotic models, (iii) integral models. The thin film asymptotic model derived by Frenkel [4] for the evolution of the film thickness is valid for the coating flow wherein the fibre radius a is much larger than the film thickness h. This model has been widely used in the investigations on the nonlinear dynamics of the coating flow on fibers at small Reynolds numbers of $Re \sim \mathcal{O}(1)$ for $h \ll a$ [5,6]. Kliakhandler et al. [7] conducted experiments in which the film thicknesses are of the order of the fibre radii. Therefore, the previously derived thin film equations under the assumption of $h \ll a$ do not apply there. Kliakhandler et al. [7] have proposed a long-wave asymptotic model which does not rely on the previously made thin-film assumptions to investigate the dynamics of the problem at $Re \sim \mathcal{O}(1)$ for the case where the film thickness is larger than the fibre radius. Craster and Matar [8] derived a new evolution equation similar to that used by Kliakhandler et al. [7] to revisit the same problem in which the radius of the free surface is much smaller than its characteristic capillary lengthscale. The numerical solutions of this equation yield information in terms of interfacial profiles, droplet spacings and

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velocities which is in excellent agreement with the three typical flow regimes in the experimental observations by Kliakhandler et al. [7].

Ruyer-Quil et al. [9] formulated an integral model in terms of the film thickness h and flow rate q using a weighted residual approach. This model accounting for inertia and streamwise viscous diffusion is valid for moderate Reynolds numbers, both small and $\mathcal{O}(1)$ aspect ratios of h/a. Comparisons between the numerical and experimental results show good agreement in both linear and nonlinear regimes.

In many practical processes, the fibre-coating is often operated in a cooling environment, for example, the glass manufacturing and dry cooling of thermoelectric power plants. In glass manufacturing process, glass fibres are made by drawing molten glass through an array of small diameter bushings. The fibres are sprayed with water from atomizing nozzles to enhance the heat removal from them [10].

The effect of thermocapillarity on the dynamics of thin films on cylinders have given rise to broad scientific interest for its technological importance. Liu and Liu [11] studied the axisymmetric long wave stability of thin film flowing down a uniformly heated vertical fiber. The results showed that the Marangoni instability and the Rayleigh-Plateau instability reinforce each other. With the increase of the thermocapillary effect, the coating flow has a tendency to break up into smaller droplets. Recently, Liu et al. [12] studied the Marangoni effect on the absolute and convective instabilities of the coating flow on a fibre. The numerical results showed that for various Marangoni number (Ma), breakup of the film mainly occurs in the absolutely unstable regimes.

Dávalos-Orozco and You [13] performed a linear stability analysis on the three-dimensional thermocapillary instability of a coating flow on a cylinder in the absence and in the presence of gravity. It was found that in the absence of gravity the thermocapillarity is possible to excite a non-axisymmetric unstable mode. Moctezuma-Sánchez, and Dávalos-Orozco [14] studied the non-axisymmetric long wave instability of a thin viscoelastic liquid film flowing down a vertical heated cylinder. The results showed that, in comparison with the Newtonian case, it is easier to excite the azimuthal modes when viscoelasticity and thermocapillarity destabilize at the same time. Recently, Ding and Wong [15] studied the three-dimensional dynamics of thin liquid films on vertical cylinders with Marangoni effect. Nonlinear simulation of the thin film model revealed that symmetry-breaking phenomenon can occur when the Marangoni number exceeds a critical value.

The theoretical development of thermocapillary instabilities in thin films on cylinders has been limited so far to films uniformly heated by the cylinders. As a temperature gradient is applied perpendicular to the cylindrical wall, there can be a purely conductive base state in which the thermocapillarity does not influence the bulk flow. However, in some practical situations the imposed temperature gradient will have a component parallel to the free surface so that additional interfacial motions are generated. The thermocapillarity involves a shear flow in the bulk at the base state. Except the thermocapillarity, an alternative way to influence the interfacial shear stress of the coating film is to apply a countercurrent gas flow. Grünig et al. [16] performed experiments on the liquid flow on a vertical wire in a countercurrent gas flow. The authors found that when the film is subject to a significant counter current gas flow the flow pattern changes, but the liquid hold-up and the interfacial area are hardly influenced. Zeng et al. [17] performed experimental studies on a promising structure, Direct-contact Liquid-on-String Heat Exchanger (DILSHE), used in dry cooling of thermoelectric power plants. In the situations where the coating film is cooled by a counter flowing gas stream, the Rayleigh-Plateau instability is modified by the interfacial shear stress induced by the countercurrent gas flow as well as the thermocapillary stress. The authors investigated the relationship between flow characteristics and heat transfer effectiveness for different combinations of the air velocities, liquid mass flow rates, and nozzle radii.

The linear stability of the thermocapillary flow with a temperature gradient along the axial direction has been investigated for plane film flows [18]. However, a careful look at previous publications indicates that the studies on the thermocapillary effect induced by an axial temperature gradient in a coating flow on a fibre are rare. The main objective of the present paper is to investigate how thermocapillarity induced by axial temperature gradients influences the linear stability and the nonlinear dynamics. The related works to the present problem are the studies on the effect of the thermocapillarity on the capillary instability of liquid jets [20] which is a fundamental problem found in several applications, for example ink-jet printing, spraying of liquids for cooling, and long liquid bridge [21] which is related to experiments on float-zone crystal growth.

The present paper is organized as follows. In Section 2, the mathematical formulation of the physical model is presented. In Section 3, we present the results and discussion. In Section 4, we summarize the results and present the conclusions.

2. Mathematical formulation

2.1. Governing equations

As shown in Fig. 1, a Newtonian fluid, of constant viscosity μ and density ρ, flows down a vertical fibre of radius $r = a$ under gravity g. The initial radius of the fluid ring measured from the centre of the fibre is $r = R$. We make an assumption that the flow is axisymmetric. A constant temperature gradient $\mathrm{d}T/\mathrm{d}z = b, b > 0$ is imposed along the vertical direction in the liquid layer. The temperature at the surface of liquid ring is denoted by T_i.

The dynamics of the flow are governed by the continuity equation, the Navier-Stokes equations and the energy equation. Assuming axisymmetric flows without any variation in the azimuthal θ-direction and without the azimuthal velocity component, the equations of motion are:

$$u_r + \frac{u}{r} + w_z = 0, \tag{1}$$

$$u_t + uu_r + wu_z = -\frac{p_r}{\rho} + \frac{\mu}{\rho}\left[u_{rr} + \frac{u_r}{r} - \frac{u}{r^2} + u_{zz}\right], \tag{2}$$

$$w_t + uw_r + ww_z = g - \frac{p_z}{\rho} + \frac{\mu}{\rho}\left[w_{rr} + \frac{w_r}{r} + w_{zz}\right], \tag{3}$$

$$T_t + uT_r + wT_z = \kappa\left[T_{rr} + \frac{T_r}{r} + T_{zz}\right], \tag{4}$$

where t denotes time, u and w denote the radial (r) and axial (z) velocity components, p denotes the pressure, T denotes the temperature, κ denotes the thermal diffusivity. Note that unless stated otherwise, the subscript denotes partial differentiation.

At the fibre surface ($r = a$), no-penetration and no-slip conditions for the velocities are

$$u = w = 0. \tag{5}$$

The fibre wall is an insulated rigid wall,

$$T_r = 0. \tag{6}$$

In the present problem, we assume the liquid is non-volatile. Thus, the effects related to evaporation, for example the phase