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Effect of gravity on the stability of viscoelastic thermocapillary liquid layers

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ABSTRACT

Thermocapillary convection appears in many polymer processing operations. In order to understand the effect of gravity for its stability, three-dimensional linear stability analysis is performed for a viscoelastic fluid. The critical Marangoni number is derived as a function of Bond number, Prandtl number and elasticity. When the Prandtl number is large, the increasing of gravity effect often makes the flow more stable, and the coupling of gravity to elasticity and thermocapillary force excites many different kinds of preferred modes. For small Prandtl number, the flow is always destabilized by gravity effect. The work done by gravity becomes a new energy source for perturbation. However, it can be either positive or negative, which is not directly related to the variation of critical Marangoni number with gravity. The effect of gravity on the instability mechanism and the properties of preferred modes are demonstrated for different elasticity.

1. Introduction

Thermocapillary flow refers to the surface-tension-driven convection in the fluid layer submitted to a horizontal temperature gradient. It has been an active topic of research for its great practical importance in many industrial applications, such as fusion welding [1] and crystal growth techniques [2]. In the theoretical studies, the model proposed by Smith &Davis [3] has been widely used, where the fluid layer above an infinite flat plane is set in motion by the a horizontal temperature gradient on the surface. The oblique hydrothermal waves predicted in their work have been observed both in the experiment [4] and numerical simulation [5].

This model has also been used in the study of thermocapillary flow in the presence of gravity. Garr-Peters [6] has performed linear stability analysis for surface-tension driven fluid layer subject to buoyant forces. The free surface is facing either upward or downward while the Prandtl numbers considered include $0.01 \leqslant Pr \leqslant 10$. It was shown that the gravity destabilizes the flow for small Prandtl numbers. Parmentier et al. [7] have examined the stability of coupled buoyancy and thermocapillary driven convection in thin fluid layers for $0.01 \leqslant Pr \leqslant 7$. The presence of travelling rolls is exhibited. Mercier and Normand [8] have studied the linear stability of buoyant-thermocapillary liquid layers for $Pr = 7$. The transition between stationary and oscillatory modes found in the experiment of Daviaud and Vince [9] is observed when the heat transfer at the free surface is introduced. Chan and Chen [10] have carried out linear stability analysis of the thermocapillary fluid layer with the effect of gravity included for $Pr = 13.9$. The results compare favorably with the experiment conducted by Riley and Neitzel [4]. The critical Marangoni number increases with Grashof number (a measure of gravity) while the preferred mode changes from oblique wave to transverse wave.

It should be noted that the model used above is very different from the Benard-Marangoni convection [11] although gravity and thermocapillary forces exist in both of them. For the latter, the fluid layer is heated from below. However, for the former, the bottom has zero heat flux. A horizontal temperature gradient is imposed on the fluid surface, and there is an inclined temperature gradient with both a horizontal and a vertical component in the fluid layer.

The thermocapillary flows for polymer liquids have also received much attention for its applications in film coating [12], drying of polymer solution [13,14], dewetting [15] and polymer processing [16,17]. It is worth noting that polymer liquids are often viscoelastic fluids, whose flow properties are very different from those of Newtonian fluids. The effect of elasticity should be considered in the study of thermocapillary flows for polymer liquids.

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A few investigations have been undertaken to study the stability of viscoelastic thermocapillary liquid layers. The influence of thermocapillary forces on the buoyancy-driven viscoelastic fluid layer has been demonstrated by Kaloni & Lou with Prandtl numbers 10 and 100 [18]. The thermocapillary force is far less than the buoyancy force and the perturbations they considered are mainly transverse and longitudinal modes. Davalos-Orozco & Chavez [19] have performed two-dimensional linear stability analysis in small wave number approximation for thermocapillary convection in a viscoelastic fluid layer under a horizontal temperature gradient. Tong et al. [20] have investigated the thermocapillar instability of a two-dimensional viscoelastic planar liquid sheet in surrounding gas. Hernández and Dávalos-Orozco [21] have examined the linear viscoelastic thermal Marangoni convection. Gravity has been neglected and the system is heated from the lower wall. The competition between stationary and oscillatory convection is shown. Hu et al. [22] have studied the instability of thermocapillary liquid layers for Oldroyd-B fluid. Three kinds of preferred modes are found in different elasticity: oblique wave, streamwise wave and spanwise stationary mode. For the first, the flow is stabilized by elasticity. The second has several fluctuations in vertical direction. The last becomes the preferred mode when the elasticity is high enough.

The impact of elasticity on flow stability has been demonstrated in the above works. However, the ground experiment and processing of polymer liquids are often in a normal gravity environment. The works for thermocapillary convection of Newtonian fluid suggest that the gravity has a significant impact on the flow stability [4,10]. Thus, the clarification of gravity effect in viscoelastic thermocapillary convection is also needed, which is the purpose of this paper.

In the present work, the study of gravity effect on the stability of thermocapillary convection has been extended from Newtonian fluid [10] to viscoelastic fluid. The Oldroyd-B fluid is applied, which is a viscoelastic model widely used for dilute solutions of macromolecules. In order to show the coupling effect of gravity, elasticity and thermocapillary force, the flow at several Bond numbers, elastic numbers and Prandtl numbers are examined. The comparisons are made with the case without gravity [22] and the effect of gravity on the instability mechanism is discussed.

2. Problem formulation

We consider the model of thermocapillary liquid layers presented by Smith and Davis [3], where the fluid on an infinite wall is set in motion by the temperature gradient on the free surface. The liquid is in contact with an inviscid atmosphere, and the gravity is imposed in the vertical direction, see Fig. 1. Here a horizontal temperature gradient is imposed on the surface of liquid layer and the bottom has zero heat flux. Due to heat transfer, there is also a vertical temperature gradient in the layer. We suppose that the temperature of basic flow T_0 is linear in x as

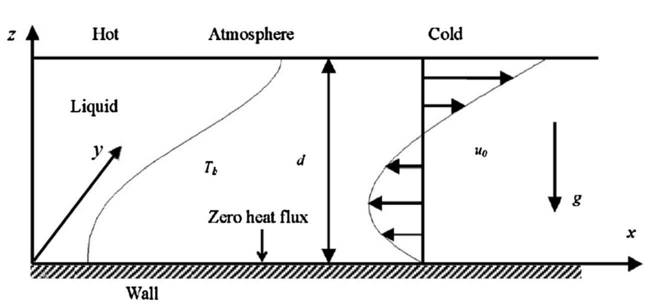

Fig. 1. Schematic of thermocapillary liquid layers in the gravity field.

imposed plus a vertical distribution T_b. The basic flow is assumed to be parallel and u_0 is the velocity. Here, d is the depth of the layer, x is the streamwise direction, and z is the wall-normal direction. For simplicity, we only consider the case related with Ref. [3], where the surface tension is big enough so that the liquid surface is non-deformable.

This is an approximate model for the convection in a fluid layer with an upper free surface in a long tank, and a temperature difference is maintained between the two endwalls [10]. The relative change of temperature in the liquid layer is assumed to be small. This is true in Ref. [4] for the experiment of silicone oil, where the temperature difference between the two endwalls is less than 10 °C while the reference temperature of the experiment is about 25 °C. Therefore, the relative changes of density, surface tension, and dynamic viscosity are also small. The variation of density with temperature leads to the buoyancy effect in the presence of gravity, and can be measured by Bond number. The variation of surface tension with temperature leads to the thermocapillary effect and can be measured by Marangoni number. These two dimensionless parameters are considered during the numerical process in the following. However, the variation of dynamic viscosity with temperature is neglected.

2.1. Governing equations

In Ref. [10], the surface tension and density of a Newtonian fluid in the gravity field are both assumed to vary linearly with the temperature. The critical parameters of buoyant-thermocapillary flow predicted by linear stability analysis in Ref. [10] are comparable with the experiment [4]. This suggests that the linear relation between the surface tension and temperature is still appropriate for the flow in the presence of gravity. For viscoelastic fluid, it has been observed in the experiments that the surface tension for polymer liquid is also linear with the temperature over limited temperature domains [23]. Therefore, we assume that the surface tension $\tilde{\sigma}$ is related to the temperature $\tilde{T}$ as $\tilde{\sigma} = \tilde{\sigma}_0 - \gamma(\tilde{T} - \tilde{T}_0)$, where γ is the negative rate of change of surface tension with temperature. The Reynolds number R is defined as $R = \frac{\rho U d}{\mu}$, where ρ, μ are the fluid density, and viscosity, respectively. U is the characteristic velocity defined as $U = b\gamma d/\mu$, b is the temperature gradient on the surface. The characteristic temperature is bd. The Marangoni number is defined as $Ma = b\gamma d^2/\mu\chi$, χ is the thermal diffusivity. There is a relation between Ma and R: $Ma = R \cdot Pr$, where $Pr = \frac{\mu}{\rho\chi}$ is the Prandtl number. The fluid density depends on temperature with the form $\rho = \rho_0[1 - a(\tilde{T} - \tilde{T}_0)]$, where a is the thermal expansion coefficient. The gravity effect is measured by the dynamic Bond number $Bo = \frac{\rho g a d^2}{\gamma}$, and g is the gravitational acceleration. These definitions are the same as those in Ref. [4].

In the presence of gravity, the distribution of fluid density will induce the buoyancy force in the vertical direction. Within Boussinesq's approximation, the dimensionless governing equations are given below [6], which are the continuity equation, the momentum equation and the energy equation, respectively.

$$\nabla \cdot \mathbf{u} = 0, \tag{2.1}$$

$$R\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla \cdot \mathbf{Q} + Bo \cdot T\mathbf{e_z}, \tag{2.2}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{Ma}\nabla^2 T. \tag{2.3}$$

Here $\mathbf{u}, p, T$ are the velocity, pressure and temperature, respectively.

For simplicity, the variation of dynamic viscosity with temperature is neglected in (2.2), which is similar to the previous works