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# Heat conduction investigation of the functionally graded materials plates with variable gradient parameters under exponential heat source load

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ABSTRACT

This paper adopted the hybrid numerical method to research the heat conduction of functionally graded materials plates (FGM plates) with variable gradient parameters under the exponential heat source load. Based on the heat balance equation, hybrid numerical method theory was established through the method of weighted residual, which considering the convective heat transfer boundary condition. Then Fourier transform and inversely Fourier transform were applied to the equation and obtained the temperature distribution of the FGM plates with variable gradient parameters. The results show that the different position temperature changing is consistent with heat source. The influence of the heat source in model is only partial, the distribution of the temperature is gradually reducing with the distance away from the heat source, and it tends to zero in infinite distance position. The change of the gradient parameter in a certain range has a great influence on the heat conduction, when the gradient parameters are maximization and minimization, the heat conduction becomes the heat conduction process of pure materials.



## 0. Introduction

Functionally graded material (FGM) [1,2] is a newly development material in recent years. Its distinguishing feature is that the composition of the material presents a continuous gradient change in the direction of thickness. The feature and function of the material also appear a gradient change. And the thermal stress and residual stress of material can be relieved. Compared with the traditional materials, the property of FGM is improved significantly. Due to its superior physical properties, it is widely used in the high temperature environment. So to analysis the distribution characteristics of the temperature and the characteristics of heat conduction is especially important.

In recent years, a large number of domestic and abroad scholars have studied the heat conduction of FGM, they are mainly focusing on the analytic method and numerical method [3]. Eskandari used a semi-analytical solution based on multi-layered approach, and present the solutions of temperature, displacements, and transient thermal stresses in functionally graded circular hollow cylinders subjected to transient thermal boundary conditions [4]. Hosseini studied the transient heat conduction in a cylindrical shell of functionally graded material by using analytical method. The shell is assumed to be in asymmetric conditions. The temperature distribution is derived analytically by using the Bessel functions [5]. Shao adopted series method of differential equations, the analytic solution for temperature and thermal/mechanical stress of a functionally graded hollow cylinder with finite length were derived [6,7]. Ebrahimi studied the thermomechanical vibration analysis of FG nanobeams subjected to in-plane thermal loads are carried out by presenting a Naiver-type solution and employing a semi-analytical differential transform method (DTM) for the first time [8–10]. Then Ebrahimi proposed a higher-order shear deformation beam theory and then the free vibration behavior of magneto–electro–therm-elastic functionally graded nanobeams is investigated and solved applying analytical solution [11,12]. Ansari present an exact solution for the nonlinear forced vibration analysis of nanobeam made of FGM subjected to thermal environment including the effect of surface stress [13]. Due to the nonlinear equations, the analytical solutions are too difficult to get for the complex heat conduction problems. Thus the numerical method has been widely applied in the heat conduction problems of FGM. Chareonsuk put forward a robust unstructured control-volume finite element method for the solution of two-dimensional transient heat conduction in functionally graded materials with isotropic properties. And explored thermal stress analysis for functionally graded materials

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at steady state with the unstructured mesh capability for arbitrary-shaped domain [14,15]. Li applies the multiple reciprocity boundary face method to solve transient heat conduction problems of functionally graded materials [16]. Zhang considered a Green's function approach to the solution of nonlinear transient heat transfer problem in a FGM object for the purpose of both the numerical analysis accuracy and computational efficiency [17]. Yu presented a new method that is formed by the differential transformation method and the redial integral boundary element method to solve transient heat conduction of functionally graded materials [18]. Ansari reported on the thermal instability of FG annular microplates with different boundary conditions. Then aimed at developing a nonclassical Mindlin rectangular functionally graded material (FGM) microplate based on the strain gradient theory (SGT) to study the thermal buckling behavior of microplates with different boundary conditions [19,20]. Ghayesh employed the modified version of the couple stress-based theory, the effects of nonlinear vibrations of the micro system and the nonlinear size-dependent dynamics of a functionally graded are investigated [21,22]. Then the modified couple stress theory along with the Mori–Tanaka homogenisation technique is employed to investigate the size - dependent oscillations of a third-order shear-deformable, the nonlinear forced vibration characteristics, the nonlinear size-dependent forced vibrations of FG model [23–25]. A special attention is paid on the effects of the material gradient index, the imperfection amplitude, and the length-scale parameter on the system dynamical response [26].

For overcoming the limitation of calculation rate of the numerical method. More numerical methods are investigated. Mesh-less method is considered to be a more potential and widely used numerical method [27]. Sladek proposed an advanced computational method for transient heat conduction analysis in continuously non-homogeneous FGM based on the local boundary integral equations with moving least square approximation of the temperature and heat flux [28]. Wang developed a mesh-less numerical model for analyzing transient heat conduction in non-homogeneous FGM, and the temperature at any point can be easily computed using the results of fictitious sources and interpolation coefficients [29]. Peng used a hybrid numerical technique to investigate the annular fin with temperature-dependent thermal conductivity, which combined the differential transformation and finite difference method [30]. Cheng presented a hybrid differential transform/control-volume method to solve hyperbolic heat conduction problems [31].

The analytical solution is very difficult to obtain, while the precision of the solution will be declined for numerical method for FGM heat conduction problems, and it also requires a lot of computational memory while using numerical method. This study initially applies hybrid numerical method into functionally graded materials plates with variable gradient parameters, which combines the accuracy of analytical method with the efficiency of numerical method. The paper obtained the distribution of temperature on the surface and further investigated the relationship between the gradient parameters and temperature distribution in model. It provides guidance for the application of FGM used in the high temperature environment. In the future research, the hybrid numerical method will be applied to study the thermoelasticity problems of functionally graded materials with variable gradient parameters. It will provide a new way for solving the thermoelasticity problems of FGM with variable gradient parameters.

## 1. Mathematical model of FGM heat conduction problem

Considering a FGM plates, it is infinite in $x, y$ directions and dispersed N elements in $z$ direction. The thermal physical property is continuous changed at each adjacent element and in each element three nodal surfaces is considered. That is called upper nodal surface, middle nodal surface and lower nodal surface, then the discrete elements are changed into nodal surface and in each surface there is a calculation temperature $T$.

Assuming that FGM model temperature distribution is zero at the initial time. The model is heated by line heat source throughout the $y$ direction after the initial time. Therefor, the heat conduction equation of FGM model can be simplified as two-dimensional heat conduction problem related to $x$ direction and $z$ direction. The heat source is located at the 1st surface of $x = 0$. The temperature value of heat source is $T_f$, the heat transfer coefficient between the upper surface and surrounding medium is $\alpha$. The simplified model is shown in Fig. 1.

Considering the internal heat source, the differential equation of two-dimensional heat conduction in the element is expressed as follows

$$\gamma_0 \frac{\partial^2 T}{\partial x^2} + \gamma_0 \frac{\partial^2 T}{\partial z^2} + g = \eta\rho \frac{\partial T}{\partial \tau} \tag{1}$$

where $\gamma_0$ is the heat conductivity, $g$ is the volume heat generation rate, $\eta$ is the material specific heat, $\rho$ is the mass density, $\tau$ is computing time.

The convective heat transfer boundary condition is taken as

$$\gamma_0 \frac{\partial T}{\partial z}|_\Gamma + \alpha(T - T_f)|_\Gamma = 0 \tag{2}$$

in which $T_f$ is the temperature of the heat source, $\alpha$ is the heat transfer coefficient, and the $\Gamma$ denotes the boundary surface.

The initial temperature is zero, so it can be denoted as

$$T|_{\tau=0} = 0 \tag{3}$$

## 2. Heat conduction theory of FGM plates based on hybrid numerical method

### 2.1. Hybrid numerical method of FGM under exponential heat source

The temperature field within a layer element is approximated as

$$T(x, z, \tau) = N(z)\phi(x, \tau) \tag{4}$$

where $N(z)$ is the matrix of shape functions, and here the quadratic shape function is used as

$$N(z) = \left[\,(1 - 3\bar{z} + 2\bar{z}^2) \quad 4(\bar{z} - \bar{z}^2) \quad (2\bar{z}^2 - \bar{z})\,\right] \tag{5}$$

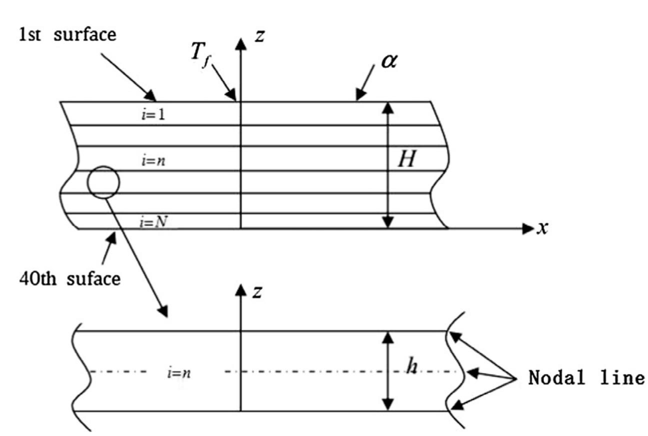


**Fig. 1.** FGM plates heat conduction problem analysis model.