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Heat and fluid flow of water and ethylene-glycol based *Cu*-nanoparticles between two parallel squeezing porous disks: LSGM approach

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ABSTRACT

This study is dedicated to analyze the heat transfer and flow of ethylene glycol and water based copper (*Cu*) nanoparticles between two squeezed parallel disks with suction/injection effects. The lower disk is assumed to be permeable. Additionally, we have considered the influence of MHD to keep the metallic particles in charge. These particles are normal to the surface and strongly effected by magnetic field. Constructed mathematical model consist of system of partial differential equations in cylindrical coordinates, based upon momentum and energy equations. The governing equations reduced to a nonlinear set of ordinary differential equations. The said set of nonlinear equations consists of squeezing number S, Hartmann number (M), nanoparticle volume fraction ϕ and suction/injection parameter (A) tackled by least square Galerkin method (LSGM). The outcomes are analyzed by means of temperature and velocity profiles for every *Cu*-water and *Cu*-ethylene glycol nanofluids. The heat transfer and flow behavior at the surface are studied via graphical plots for local Nusselt number and skin friction. It is observed that local Nusselt number achieved from *Cu*-water remain lesser than *Cu*-ethylene glycol while the behavior for skin friction coefficient is totally opposite. We support our theoretical study via a detailed evaluation of outcomes. The obtained results via least square Galerkin method (LSGM) are compared with RK (order-4) and already existing results. Moreover, graphical representation, the error, convergence and comparison analysis of outcomes endorsing that the least square method is extremely effective. The suggested method could be extended to other nonlinear problems.

1. Introduction

Squeezing is an important phenomenon that is used in the various mechanical and industrial processes such as squeezing flows, two parallel disks or plates compress the material between them via a reverse directional motion and resultantly extent the flow in the direction of radial axis. The common use of these flows found in various engineering applications, such as molding and compression of polymers and metals, hydraulic systems and lubrication systems. In viscometers, the rheological nature of fluid can be measure by squeezing [1]. The phenomena of squeezing flow between plates has a wide class of applications in the transfer of heat and other board components. The rump of the chip attached to a heat sink and then it is pushed counter to the board to generate the squeezing flow between disks/plates [2]. The non-Newtonian coolants are used to fulfill this purpose due to vital class of application systems. Many authors published a number of researcher articles on squeezing flows and a comprehensive literature is available related to this domain. In lubrication systems, the squeezing flow firstly reported by Stefan [3]. Engmann [4] provided a review on squeezing flow and its advances after the ground-breaking work of Stefan. It is precised that the squeezing flow have transient nature and the experimental explanations are quiet incompetent to propose a single generalize relation to envisage the behavior of flow as fundamental rheological measure function. One can find some study related to squeezing flows in [5–7]. The influence of wall suction and MHD brings positive response to the flow field and rise the detection chance on the senor surface because of squeezing. The devotion of many authors related to this domain can be found in [8–11]. Recently, Mohyud-Din et al. [12] studied heat transfer for squeezing flow of a Casson fluid. They found that increasing values of squeezed number (S) and Casson fluid parameter (β) magnitude of Nusselt number is decreasing function while it is an increasing function for enhanced values of Prandtl and Eckert numbers.

In many processes cooling/heating is an essential part of extrusion and molding. One element, which took the heat exchange pro-

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gression to new stature, is the consolidation of nanoparticles inside the ordinary liquids. The metals have higher thermal conductivity when contrasted with the usually utilized fluids. Resultantly the thermal conductivity of base liquid comprising the metal particles is more prominent than the ordinary base fluids. Nevertheless, the extent of metal particles can be vital, as the expansive particles can settle down under the impact of gravity because of higher molecule weight. Variation of thermal conductivity and liquid viscosity by dispersing ultra-fine particles exposed by Masuda et al. [13]. After the prominent effort by Choi [14], a new field seemed as nanofluids. Choi introduced a new term thermo-fluid for nanofluids wherein solid-liquid mixture having base fluid and solid nanometer-sized particles named as nanoparticles. Glycol-built nanofluids comprising nanoparticles of copper and thermal conductivity special effects investigated by Eastman et al. [15]. Buongiorno [16] is an efficient and useful nanofluid model. He considered the slip effects between the nanoparticles and molecules of base fluids. He concluded that thermophoresis and Brownian motion are important slip effects. Later, Mohyud-Din et al. [17] analyzed the transfer of heat and mass of nanofluids arising in biosciences by means of Buongiorno's model. The slip and homogenous models are invented in parallel to examine the nanofluid flows. A comprehensive study related to nanofluids domain available in the literature by means of various aspects [18–24]. The cited work shows essential properties of nanofluids because of their usage in heat exchanger, packing, extrusion and coolants processes.

The presence of tiny particles inside base fluids studied by various authors. It is reported that these small particles enhanced the thermal conductivity of the base fluids. The influence of micro rotation for the nanofluid flow studied Hussain et al. [25]. They observed that the presence of micro-rotation increase the effect on the heat transfer coefficient while decrease the effect on the skin friction coefficient. Moreover, they found that Copper (Cu), Ag-kerosene oil provide higher heat transfer rate than Cu, Ag-water. The nanoparticles can have various shapes and sizes. The Ag, Cu shape is like CNTs (i.e. single & multiple Carbon nanotubes) while spherical shaped are called metallic nanoparticles. Later on, many scholars used various type of base fluids to study nanofluid flows. One can find some recent literature in [26–29]. Recently, Haq et al. [30] investigated the flow and heat transfer of ethylene glycol and water based Cu-nanoparticles between two parallel disks besides the influence of suction/injection. The cited study in our work is endorsing that the theoretical investigation is very important because it saves both time and money. The development or extension of numerical or analytical methods is a problem of interest for authors. Previously, various authors proposed or modified both (analytical and numerical) algorithms [31–33]. This motivation is due to the solution of nonlinear problems. Recently, the least square method is used by various authors [34–40].

On the bases of above mentioned literature, we have developed the heat transfer and flow model of ethylene glycol and water based copper (Cu) nanoparticles between two squeezed parallel disks besides the influence of injection/suction effects. Moreover, MHD effect is also introduced to control the metallic particles behavior. These particles are normal to the surface and strongly effected by magnetic field. The lower disk is assumed to be permeable. The governing equations reduced to a nonlinear set of ODEs. The set of nonlinear ODEs tackled via least square Galerkin method (LSGM). The outcomes are analyzed by means of temperature and velocity profiles for every Cu-water and Cu-ethylene glycol nanofluids. The heat transfer and flow behavior at the surface are studied via graphical plots for local Nusselt number and skin friction. Moreover, physical changes also presented. It is observed that local Nusselt number achieved from Cu-water remain lesser than Cu-ethylene glycol while the behavior for skin friction coefficient is opposite. Conclusion is drawn based on whole investigation and differentiates the heat transfer and flow behavior for each mixture.

2. Mathematical model

The MHD incompressible ethylene glycol and water based flow of nanofluid between two parallel infinite disks has been supposed in such a way that the distance between them is finite. Moreover, it is assumed that the phenomena is based on two base fluids (ethylene glycol and water) saturated with copper nanoparticles. A magnetic field $B_0\left(\sqrt{(1-\alpha t)^{-1}}\right)$ is applied in the normal direction of the disks while the induced magnetic field is negligible because of low Reynold number. The constant temperatures (T_h and T_w) are defined at the upper surface of the channel at $z = h(t)$ and lower surface at $z = 0$ respectively. Furthermore, it is supposed that upper disk is in motion having velocity $aH\sqrt{(1-at)^{-1}}$ and lower disk is permeable besides the suction/injection velocity $w = W_0$. The motion of the upper disk is in opposite direction of lower disk located at $z = 0$. The cylindrical coordinate system (r, α, z) is assumed. The rotational symmetry of the flow $(\partial/\partial\alpha = 0)$, v that is azimuthal component of velocity $V = (u, v, w)$ disappears identically. Therefore, the energy and governing equations for the two-dimensional unsteady and viscous fluid flow takes the form as below [30]:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma}{\rho_{nf}}B^2(t)u, \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial z} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right), \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right), \tag{4}$$

In above Eqs. (1)-(4) the velocity component in r and z-direction represented by u and v correspondingly. The terms T, p, k_{nf}, μ_{nf} and ρ_{nf} are represented by temperature, pressure, thermal conductivity, dynamic viscosity and density of nanofluid which given as [30]:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\ \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,\ v = \frac{\mu_{nf}}{\rho_{nf}},\ \alpha_{nf} = \frac{k_{nf}}{(pc_p)_{nf}},$$

$$\frac{k_{nf}}{k_f} = \frac{(2k_f + k_s) - 2\phi(k_f - k_s)}{(2k_f + k_s) + \phi(k_f - k_s)},\ (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s.$$

In above expressions thermal conductivity of base fluid, nanoparticle volume fraction, specific heat capacity and thermal conductivity of the solid fraction are denoted by $k_f, \phi, \rho c_p$ and k_s respectively. The boundary condition subject to the problem (1)–(4) are given as:

$$u(z) = 0,\ w(z) = -W_0,\ T(z) = T_w, \quad \text{as } z \to 0, \tag{5}$$

$$u(z) = 0,\ w(z) = \mathrm{d}h/\mathrm{d}t,\ T(z) = T_h, \quad \text{as } z \to h(t), \tag{6}$$

where T_w and T_h specify the temperature at lower and upper disks respectively. In order to convert the system (1)–(6) into