International Journal of Heat and Mass Transfer 123 (2018) 994–998

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Temperature-dependent anisotropic bodies thermal conductivity tensor components identification method

V.F. Formalev *, S.A. Kolesnik

Moscow Aviation Institute (National Research University), Moscow, Russia

ARTICLE INFO

Article history:
Received 24 December 2017
Received in revised form 18 February 2018
Accepted 5 March 2018
Available online 20 March 2018

Keywords:
Anisotropic bodies
Inverse coefficient problems
Regularizing functional
Thermal characteristic temperature dependences
Parametric identification method
Conjugate problems

ABSTRACT

This paper describes a new method of solving of inverse coefficient thermal conductivity problems in anisotropic bodies directed on the identification of temperature dependences of thermal conductivity tensor components. This method includes the following: quadratic residue construction between testing and theoretical temperature values, minimized gradient descent implicit method, parametric identification method, construction and numerical solution of conjugate problems relating to anisotropic thermal conduction, regularizing functional development based on prior assumptions on smoothness of temperature functions of thermal conduction components of anisotropic bodies, permitting to increase the whole method stability. Basing on this method many results were found relating to the identification of thermal conduction tensor components depending on temperature in the form of practically arbitrary functions: monotonous functions, having minimum and maximum points, flex points, etc.

1. Introduction

In the process of solving of direct problem of the thermal conduction theory the temperature fields (consequences) are determined by causal links – difference equations, initial-boundary conditions, different coefficients. In the process of solving of identification problem (inverse problems) conversely – these causal links (boundary conditions, coefficients, differential equations, etc.) are determined according to the consequence (mostly on temperature fields). Besides, there are hundreds and thousands of publications relating to thermal conduction direct problem solution in isotropic areas, particularly, monographs and books Kartashov [1], Zarubin [2], Zarubin and Kuvyrkin [3], Lykov [4], Carslaw and Jaeger [5], and also in anisotropic mediums Formalev [6], in articles Zarubin, Kuvyrkin and Savelyeva [7], Kartashov [8], Formalev and Kolesnik [9–11]. On inverse problems of thermal conductivity theories in isotropic mediums we may note the following monographs: Samarskiy and Vabischevich [12], Alifanov et al. [13], Tikhonov and Arsenin [14], Beck et al. [15]. On inverse problems of thermal conductivity in anisotropic mediums we may note the following articles [16–18].

The authors do not know any publications relating to the identification of temperature dependences of tensor components of anisotropic thermal-conductivity (nonlinear mediums). This is due to the fact that standard problems in general and inverse coefficient problems in particular are incorrect, because significant result errors can correspond to small experimental data (input data) deviations, as a result, these results become inadequate. The development of methods strengthening the numerical solving algorithm stability is one of the fundamental tasks in identification problems.

Within this paper the stable method of the identification of temperature dependences of anisotropic thermal-conductivity tensors was first developed and tested on the basis of the implicit method of gradient descent of the quadratic residual functional minimization, regularizing functional construction and its inclusion into the main quadratic residual functional and efficient absolutely stable method of the numerical solving of direct and conjugate problems on sensitivity matrix identification. Discovered results confirm that this method is rather efficient.

2. Problem statement and solving method

The first initial-boundary value problem of anisotropic thermal conductivity is considered within this paper against temperature distribution $T(x, y, t)$ in rectangular field $l_1 \times l_2$, $t > 0$. All thermal conductivity tensor components $\lambda_{11}(T), \lambda_{12}(T), \lambda_{22}(T)$ depend on

* Corresponding author.
E-mail addresses: formalev38@yandex.ru (V.F. Formalev), sergey@oviont.com (S.A. Kolesnik).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.03.014
0017-9310/

temperature, constant temperature is set at the boundaries $T_{\max}$, besides, initial temperature is $T_{\min} < T(x,y,t) \leqslant T_{\max}$. The component domain coincides with the interval $T \in [T_{\min}; T_{\max}]$.

$$\frac{\partial}{\partial x}\left(\lambda_{11}(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial x}\left(\lambda_{12}(T)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\lambda_{21}(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_{22}(T)\frac{\partial T}{\partial y}\right) = c\rho(T)\frac{\partial T}{\partial t}, x \in (0; l_1), y \in (0; l_2), t > 0; \quad (1)$$

$$T(x,0,t) = T(0,y,t) = T_{\max}, T(x,l_2,t) = T(l_1,y,t) = T_{\max} \quad (2)$$

$$T(x,y,0) = T_{\min}, \quad (3)$$

The inverse problem can be formulated as follows: on experimental temperature distribution in space-time points

$$T((x,y)_i, t^k) = \widetilde{T}_{i,k}, i = \overline{1,I}, k = \overline{1,K} \quad (4)$$

using symbolic models (1)–(3) define functions $\lambda_{11}(T), \lambda_{12}(T), \lambda_{22}(T)$ in the interval $T \in [T_{\min}; T_{\max}]$.

The required functions $\lambda_{11}(T)$, $\lambda_{12}(T)$, $\lambda_{22}(T)$, will be searched as a linear combination of the following linear continuous basis functions $N_m(T)$ [16], defined on segments ΔT_m, $m = \overline{1,M}$ $(T_{\min} \leqslant T \leqslant T_{\max})$, assigned to each node $T_m, m = \overline{0,M}$, besides, its node values $T = T_m$, $m = \overline{0,M}$ are equal to one and other values are equal to zero:

$$N_m(T) = \begin{cases} 0, T < T_{m-1}; \\ \frac{T-T_{m-1}}{T_m-T_{m-1}}, T_{m-1} \leqslant T \leqslant T_m; \\ \frac{T_{m+1}-T}{T_{m+1}-T_m}, T_m \leqslant T \leqslant T_{m+1}; 0, T > T_{\max}; \end{cases} \quad m = \overline{1,M-1},$$

$$N_0(T) = \begin{cases} \frac{T_1-T}{T_1-T_0}, T_0 \leqslant T \leqslant T_1; \\ 0, T > T_1, T < T_0; \end{cases} \quad m = 0;$$

$$N_M(T) = \begin{cases} \frac{T-T_{M-1}}{T_M-T_{M-1}}, T_{M-1} \leqslant T \leqslant T_M; \\ 0, T < T_{M-1}, T > T_M; \end{cases} \quad m = M. \quad (5)$$

The functions $\lambda_{i,j}(T)$ $i,j = 1,2$ are approximately defined as linear combinations of functions (5), where the following values $\lambda_{i,j}^m$, $i,j = 1,2$, $m = \overline{0,I}$ of searching node functions T_m, due to definition are used as coefficients.

So, the coefficients $\lambda_{i,j}(T), i,j = 1,2$ are approximately defined as the following linear combinations of basic functions (5) where the coefficients $\lambda_{i,j}^m$, $i,j = 1,2$, $m = \overline{0,M}$ are searching node values of functions $\lambda_{i,j}(T)$:

$$\lambda_{11}(T) \approx \sum_{m=1}^{M} \lambda_{11}^m \cdot N_m(T), \quad (6)$$

$$\lambda_{12}(T) \approx \sum_{m=1}^{M} \lambda_{12}^m \cdot N_m(T), \quad (7)$$

$$\lambda_{22}(T) \approx \sum_{m=1}^{M} \lambda_{22}^m \cdot N_m(T). \quad (8)$$

Let's introduce desired quantity designation $\lambda_{11}^1 = \lambda_{11}(T_{\min})$; $\lambda_{12}^1 = \lambda_{12}(T_{\min})$; $\lambda_{22}^1 = \lambda_{22}(T_{\min})$; $\ldots$; $\lambda_{11}^M = \lambda_{11}(T_{\max})$; $\lambda_{12}^M = \lambda_{12}(T_{\max})$; $\lambda_{22}^M = \lambda_{22}(T_{\max})$. To determine vector components $\boldsymbol{\lambda} = (\lambda_{11}^1, \ldots, \lambda_{11}^M, \lambda_{12}^1, \ldots, \lambda_{12}^M, \lambda_{22}^1, \ldots, \lambda_{22}^M)^{\mathrm{T}}$ in Eqs. (6)–(8) we'll input the following quadratic functional:

$$S(\boldsymbol{\lambda}) = \frac{1}{2}\sum_{i=1}^{I}\sum_{k=1}^{K}[T_{i,k}(\boldsymbol{\lambda}) - \widetilde{T}_{i,k}]^2 \quad (9)$$

as sums on space-time variables of squared deviations of observed values $\widetilde{T}_{i,k}$, where input unknown (searching) parameters in the points $((x,y)_i, t^k)$, from target values $T_{i,k}(\boldsymbol{\lambda}) \equiv T_{i,k}((x,y)_i, t^k, \boldsymbol{\lambda})$, calculating on arbitrary vector values $\boldsymbol{\lambda}$.

It's assumed that in case of functional (9) stationary value attainment, the searching characteristics, input in the experimental values $\widetilde{T}_{i,k}$, will be approximately coincided with the characteristics, following which the temperature target values have been found.

In order to minimize this functional the *implicit* method of gradient descent is used

$$\boldsymbol{\lambda}^{(n+1)} = \boldsymbol{\lambda}^{(n)} - \alpha_n grad\, S(\boldsymbol{\lambda}^{(n+1)}). \quad (10)$$

where n – previous iteration number, α_n – small parametric steps following the condition $(\alpha_n > 0)$

$$S(\boldsymbol{\lambda}^{(n+1)}) < S(\boldsymbol{\lambda}^{(n)}). \quad (11)$$

The iteration process (10) is finished when the functional achieves the following stationary values $grad\, S(\boldsymbol{\lambda}^{(n+1)}) \approx 0$, i.e. when performing

$$\left|grad\, S(\boldsymbol{\lambda}^{(n+1)})\right| \leqslant \varepsilon, \quad (12)$$

where ε – required accuracy of the iteration process.

To calculate the functional gradient the function $T_{i,k}(\boldsymbol{\lambda}^{(n+1)})$, included in (9), within $\boldsymbol{\lambda}^{(n)}$ should be taken a Taylor series expansion by retention of linear components against $\Delta\boldsymbol{\lambda}^{(n)}$ [16]. We come to the following equation:

$$S(\boldsymbol{\lambda}^{(n+1)}) \approx \frac{1}{2}\sum_{i=1}^{I}\sum_{k=1}^{K}\left[\left(T_{i,k}(\boldsymbol{\lambda}^{(n)}) + \sum_{l=0}^{3M+2}\frac{\partial T_{i,k}(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_l}\Delta\lambda_l^{(n)}\right) - \widetilde{T}_{i,k}\right]^2 + O(\|\Delta\boldsymbol{\lambda}\|^2). \quad (13)$$

In this case the functional (13) gradient components are calculated by the following equations:

$$\frac{\partial S(\boldsymbol{\lambda}^{(n+1)})}{\partial \lambda_l} = \sum_{i=1}^{I}\sum_{k=1}^{K}\left[(T_{i,k}(\boldsymbol{\lambda}^{(n)}) - \widetilde{T}_{i,k}) + \sum_{l=0}^{3M+2}\frac{\partial T_{i,k}(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_l}\Delta\lambda_l^{(n)}\right] \times \left[\frac{\partial T_{i,k}(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_l} + \frac{\partial}{\partial \lambda_l}\left(\sum_{l=0}^{3M+2}\frac{\partial T_{i,k}(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_l}\Delta\lambda_l^{(n)}\right)\right]$$
$$\approx \sum_{i=1}^{I}\sum_{k=1}^{K}\left[(T_{i,k}(\boldsymbol{\lambda}^{(n)}) - \widetilde{T}_{i,k}) + \sum_{l=0}^{3M+2}\frac{\partial T_{i,k}(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_l}\Delta\lambda_l^{(n)}\right] \times \frac{\partial T_{i,k}(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_l}, \quad l = \overline{0,3M+2}, \quad (14)$$

where $3 \cdot (M+1)$ – number of desired parameters.

The functional gradient vector with components (14) can be represented in the following vector-matrix form:

$$grad\, S(\boldsymbol{\lambda}^{(n+1)}) = Z^T(\boldsymbol{\lambda}^{(n)})(\boldsymbol{T}(\boldsymbol{\lambda}^{(n)}) - \widetilde{\boldsymbol{T}}) + Z^T(\boldsymbol{\lambda}^{(n)})Z(\boldsymbol{\lambda}^{(n)})\Delta\boldsymbol{\lambda}^{(n)}, \quad (15)$$

where the upper index «T» means conjugation and Z matrix has the following view:

$$Z(\boldsymbol{\lambda}^{(n)}) = \begin{pmatrix} u^0((x,y)_1, t^1, \boldsymbol{\lambda}^{(n)})\ldots & v^0((x,y)_1, t^1, \boldsymbol{\lambda}^{(n)})\ldots & w^0((x,y)_1, t^1, \boldsymbol{\lambda}^{(n)})\ldots \\ u^1((x,y)_2, t^1, \boldsymbol{\lambda}^{(n)})\ldots & v^1((x,y)_2, t^1, \boldsymbol{\lambda}^{(n)})\ldots & w^1((x,y)_2, t^1, \boldsymbol{\lambda}^{(n)})\ldots \\ \ldots & \ldots & \ldots \\ u^{I\cdot K}((x,y)_I, t^K, \boldsymbol{\lambda}^{(n)})\ldots & v^{I\cdot K}((x,y)_I, t^K, \boldsymbol{\lambda}^{(n)})\ldots & w^{I\cdot K}((x,y)_I, t^K, \boldsymbol{\lambda}^{(n)})\ldots \end{pmatrix}, \quad (16)$$