International Journal of Heat and Mass Transfer 122 (2018) 128–137

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Semi-analytic approximation of the temperature field resulting from moving heat loads

D.W.M. Veldman [a,*], R.H.B. Fey [a], H.J. Zwart [b,a], M.M.J. van de Wal [c], J.D.B.J. van den Boom [c], H. Nijmeijer [a]

[a] Department of Mechanical Engineering, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands
[b] Department of Applied Mathematics, University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands
[c] ASML, De Run 6501, 5504 DR Veldhoven, The Netherlands

ARTICLE INFO

Article history:
Received 6 December 2017
Accepted 18 January 2018

Keywords:
Moving heat source
Finite elements
Wafer heating

ABSTRACT

Moving heat load problems appear in many manufacturing processes, such as lithography, welding, grinding, and additive manufacturing. The simulation of moving heat load problems by the finite-element method poses several numerical challenges, which may lead to time consuming computations. In this paper, we propose a 2D semi-analytic model in which the problem in two spatial dimensions is decoupled into three problems in one spatial dimension. This decoupling significantly reduces the computational time, but also introduces an additional error. The method is applied to a wafer heating example, in which the computational time is reduced by a factor 10 at the cost of a 4% error in the temperature field.

1. Introduction

Moving heat load problems occur in many manufacturing processes, such as welding [1–8], grinding [9,10], metal cutting [11,12], laser hardening of metals [13,14], and additive manufacturing [15–18]. More recently, moving heat load problems are also studied in precision engineering because of their emerging relevance in lithography systems for the semiconductor industry. Because this is still an emerging problem only a few introductory references are available [19–21]. In the lithography application, it is customary to consider a two-dimensional (2D) spatial domain (see [20,21]), whereas three-dimensional (3D) spatial domains are typical for the other applications.

The basis of the theory for moving heat sources was developed by Rosenthal [1,22], who observed that when the path of the heat load is long enough, the temperature distribution around the source soon becomes constant. Assuming constant material properties, Rosenthal developed closed-form analytic expressions for these quasi-stationary temperature fields resulting from point, line, and plane heat sources. Although Rosenthal's analysis provides valuable estimates, transient effects and position or temperature-dependent coefficients are important in many applications. In these situations the problem is solved by Finite Element (FE) analysis (see for example [2,3,16]).

Solving a moving heat load problem by the FE method poses several numerical challenges. One problem is that by fixing the coordinate frame to the heat load we obtain a convection-diffusion problem. It is well known that the FE discretization of such problems may result in spurious oscilations [23]. Spurious oscilations can be prevented in two ways. In the first approach, the mesh size in the direction of the velocity of the moving load is chosen smaller than $2D/v$, where D [m^2/s] denotes the thermal diffusivity of the material and v [m/s] denotes the velocity of the moving load [23]. Note that this approach is computationally demanding when the velocity v is high. In the second approach, upwinding schemes [23,24] are used. These schemes prevent spurious oscilations at the cost of an increased discretization error.

Another problem is that the area in which the heat load is applied is typically small. This makes both the spatial and temporal discretization of such problems computationally demanding. For example, Zhang et al. [15] report that for a Gaussian heat distribution, the mesh size should be at least twice as small as the radius of the heat distribution and at least two time steps are needed for the time that the heat load travels along one element.

Because of these considerations, many problems require a small mesh size. For a static mesh, this mesh size needs to be used in the whole region through which the heat load travels, which results in models with many Degrees of Freedom (DOFs). To keep the number of DOFs limited, adaptive meshing strategies have been

* Corresponding author.
E-mail addresses: d.w.m.veldman@tue.nl (D.W.M. Veldman), r.h.b.fey@tue.nl (R.H.B. Fey), h.j.zwart@tue.nl (H.J. Zwart), marc.van.de.wal@asml.com (M.M.J. van de Wal), joris.van.den.boom@asml.com (J.D.B.J. van den Boom), h.nijmeijer@tue.nl (H. Nijmeijer).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.01.085
0017-9310/

proposed [3,4,17], which lead to significant reduction in computational effort. Note that these schemes require some cost for updating the mesh and that the temporal discretization remains challenging, since the adaptive mesh will keep the mesh size near the source small.

For problems with constant coefficients, spatial discretization can be avoided by semi-analytic methods [5–8,25]. In these methods, the temperature field is expressed as the convolution of the fundamental solution of the heat equation and the applied heat load. The convolution over space can typically be solved analytically, so that only numerical evaluation of the convolution over time remains. This is still a computationally intensive operation when the solution is evaluated on a fine grid.

In this paper, we propose a novel semi-analytic approximation method to reduce the computational cost of 2D transient moving load problems with constant coefficients. We construct a semi-analytic approximation in which we decouple the problem in two spatial dimensions into three problems in one spatial dimension. This significantly reduces the computational cost, especially on fine grids. The proposed method is demonstrated by an example from precision engineering, more specifically for a wafer heating problem.

The remainder of this paper is structured as follows. In Section 2, the semi-analytic approximation for the temperature field is introduced on an infinite domain. In Section 3, we give a physical interpretation of the semi-analytic approximation. In Section 4, we discuss the modeling of edge effects and repetitive scanning patterns, which are typically encountered in lithography and additive manufacturing. In Section 5, we apply the developed techniques in a wafer heating example. In Section 6, the conclusions are presented and the results are discussed.

2. Semi-analytic approximation

2.1. Problem formulation

We consider heat conduction in a thin infinite plate with thickness H [m] and constant material properties (see Fig. 1). The heat losses to the surrounding media at the top and bottom of the plate are proportional to the temperature with constant heat transfer coefficients h_c^{top} and h_c^{bot} [W/m^2 K], respectively. Because the plate is thin, the temperature gradient along the thickness of the plate can be neglected. The resulting temperature field $T_{2D} = T_{2D}(x, y, t)$ [K] relative to a reference temperature T_r satisfies the heat equation, see for example [22]

$$\rho c H \frac{\partial T_{2D}}{\partial t} = kH\left(\frac{\partial^2 T_{2D}}{\partial x^2} + \frac{\partial^2 T_{2D}}{\partial y^2}\right) - \left(h_c^{\text{top}} + h_c^{\text{bot}}\right)T_{2D} + Q, \tag{1}$$

where ρ [kg/m^3] is the mass density, c [J/kg K] the heat capacity, k [W/mK] the thermal conductivity, and Q [W/m^2] the applied heat

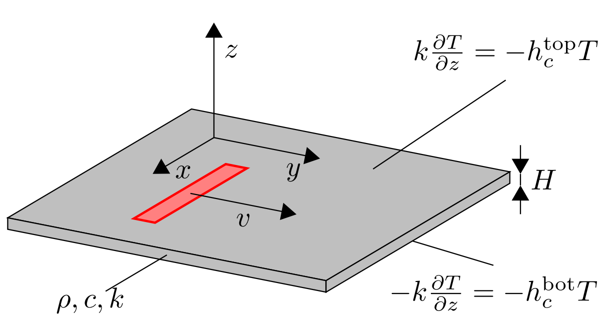

Fig. 1. The considered infinite plate.

load. We will consider (1) on the unbounded domain $(x, y) \in \mathbb{R}^2$ with zero initial conditions $T_{2D}(x, y, t = 0) = 0$.

We assume that the heat load Q is of the form

$$Q(x, y, t) = X(x)Y(y - vt)\bar{Q}(t), \tag{2}$$

where $X(x) \geqslant 0$ [1/m] describes the shape of the applied heat load in the x-direction, $Y(y) \geqslant 0$ [1/m] describes the shape of the applied heat load in the y-direction, v [m/s] denotes the velocity of the moving load and $\bar{Q}(t) \geqslant 0$ [W] is the net amount of heat applied at time t. Note that the uniform heat load applied in a rectangular area shown in Fig. 1 can be written in this form by taking block functions for $X(x)$ and $Y(y)$. Such a heat load has been considered in laser hardening [14] and will also be considered in the lithography example in Section 5. Also the Gaussian heat distribution considered in many applications (see for example [2,3,15,18]) is of the form in (2). Observe that Q moves with a constant velocity v in positive y-direction.

When we divide (1) by $\rho c H$, we obtain

$$\frac{\partial T_{2D}}{\partial t} = D\left(\frac{\partial^2 T_{2D}}{\partial x^2} + \frac{\partial^2 T_{2D}}{\partial y^2}\right) - hT_{2D} + \Theta, \tag{3}$$

where $D = k/\rho c > 0$ [m^2/s] denotes the thermal diffusivity, $h = (h_c^{\text{top}} + h_c^{\text{bot}})/(\rho c H) \geqslant 0$ [1/s], and $\Theta = Q/(\rho c H)$ [K/s] can be written as

$$\Theta(x, y, t) = X(x)Y(y - vt)\bar{\Theta}(t), \tag{4}$$

where $\bar{\Theta}(t) = \bar{Q}(t)/(\rho c H)$ [m^2 K/s].

The fundamental solution of (3) (i.e. the response of the homogeneous equation (3) with $\Theta \equiv 0$ resulting from the initial condition $T_{2D}(x, y, t = 0) = T_0\delta(x)\delta(y)$, with $T_0 = 1$ [m^2 K]) is given by

$$T_0\Phi_{2D}(x, y, t) = T_0 e^{-ht}\Phi(x, t)\Phi(y, t), \tag{5}$$

where $\Phi(x, t)$ denotes the fundamental solution of the heat equation in one spatial dimension

$$\Phi(x, t) = \frac{1}{\sqrt{4D\pi t}} \exp\left(\frac{-x^2}{4Dt}\right). \tag{6}$$

This can be checked by differentiating (5) to time and using that $\Phi(x, t)$ is the solution to the one-dimensional (1D) heat equation (i.e. $\frac{\partial \Phi}{\partial t} = D\frac{\partial^2 \Phi}{\partial x^2}$). Since we are assuming zero initial conditions, Duhamel's principle [26] asserts that

$$T_{2D}(x, y, t) = \int_0^t \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Phi_{2D}(x', y', \tau)\Theta(x - x', y - y', t - \tau)\, dx'\, dy'\, d\tau. \tag{7}$$

When we subsitute (4) and (5) in this equation, we find that

$$T_{2D}(x, y, t) = \int_0^t f(y, t, \tau)N(x, \tau)\, d\tau, \tag{8}$$

where we have introduced

$$f(y, t, \tau) = \int_{-\infty}^{+\infty} e^{-h\tau}\Phi(y', \tau)Y(y - y' - v(t - \tau))\bar{\Theta}(t - \tau)\, dy', \tag{9}$$

$$N(x, \tau) = \int_{-\infty}^{+\infty} \Phi(x', \tau)X(x - x')\, dx'. \tag{10}$$

2.2. The approximate solution

We will introduce an approximate solution by simplifying the integral in (8). Note that the only factor in (8) that depends on x is $N(x, \tau)$. In our semi-analytic approximation, we move this factor outside the integral.