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# Gas nucleus growth in high-viscosity liquid under strongly non-equilibrium conditions

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ABSTRACT

The present paper describes the growth dynamics of a gas bubble formed as a result of nucleation in a highly-viscous gas-saturated liquid under rapid decompression. The mathematical model presented combines both dynamical and diffusion problems. The half-analytical solution for the dissolved gas concentration profile, bubble growth rate and the rate of bubble mass change is found. In contrast to previously reported solutions, it can be applied at all stages of the process in a wide range of supersaturation. The half-analytical solution is in good agreement with the results of numerical simulations and with known solutions for limiting cases.



## 1. Introduction

The underlying physics of volcanic eruptions has been a topic of active research for several decades [1]. The vivid interest to this topic is stimulated by the needs of the risk assessment in relation to potential hazards of specific volcanoes. In this context, the volcanoes with the most destructive (explosive) type of eruption are of special interest [2]. Though such eruptions are quite rare, they are highly unpredictable and most catastrophic.

Due to the lack of methods for direct measurements in the ascending magma, a continuous monitoring of an active volcano cannot provide complete information about the processes developing in the volcano conduit during eruption. The beginning of eruption itself can be predicted only from proxy indicators [3]. Analyzing the phenomena and the eruptive products observed at the surface, one can state with confidence only the presence of phase transitions (from the structure of the solidified magma samples) [4,5] and the destruction of initially continuous magma flow (from the structure of erupted magma) [6]. Therefore, development of models of the magma flows in volcanic conduits within the framework of multiphase fluid mechanics seems to be particularly relevant [7]. Such models should provide a consistent description of mechanisms that determine the type and scenario of eruption, and a sound interpretation of the field observations.

Theoretical and experimental modeling of explosive eruptions should describe the following aspects of the problem: three-phase state of magma under the action of the decompression wave at the initial stage of eruption [8]; structure of the three-phase flow of the ascending magma in the volcano conduit [9,10]; dynamics of destruction of vesiculated and partially crystallized magma, with magma fragmentation and subsequent transition to a gas suspension [11,12]; impact of magma rheology on flow structure and eruption dynamics in general [13]. All the above problems are extremely difficult for theoretical and experimental study, representing separate research directions.

There exists a rich literature on experimental and mathematical modeling of volcanic eruptions, covering the process in general as well as the sub-processes involved [14]. One of the key sub-processes is the genesis of magma cavitation in decompression waves, which predetermines in many ways the two-phase flow structure in the volcano conduit and, thereby, the form and the type of eruption [15]. In the context of the present paper, the description of nucleation kinetics and gas-bubble growth under rapid decompression of the magmatic melt represents the problem of central interest.

The history of research on cavitation in liquids encompasses more than a century, starting with the pioneering works by Rayleigh [16], Plesset [17,18], Scriven [19] and others. A comprehensive description of classical models of the bubble growth, with

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**Nomenclature**

$C$ concentration of gas dissolved in liquid, kg/kg
$\overline{C} = (C - C_f)/\Delta C_i$ nondimensional concentration
$\Delta C_i = C_i - C_f$ initial supersaturation, kg/kg
$C_s$ equilibrium concentration of gas dissolved in liquid, kg/kg
$D$ diffusion coefficient, m$^2$/s
$E^*_\eta$ activation energy for "dry" liquid, J
$k_B$ Boltzmann constant, J·K$^{-1}$
$k_\eta$ empirical coefficient
$K_H$ Henry's constant, Pa$^{-1}$
$m$ mass, kg
$m_0 = (4\pi/3)\rho_{gi}R_0^3$ characteristic bubble mass, kg
$\bar{m} = m/m_0$ nondimensional mass
$n$ parameter
$p$ pressure, Pa
$\bar{p} = (p - p_f)/\Delta p_i$ nondimensional pressure
$\Delta p_i = p_i - p_f$ value of initial decompression, Pa
**Pe** $= \frac{\Delta p_g R^2}{\eta D}$ Peclet number
$r$ radial coordinate, m
$\bar{r} = r/R_0$ nondimensional radial coordinate
$R$ bubble radius, m
$\overline{R} = R/R_0$ nondimensional bubble radius
$R_0 = \sqrt{\frac{D\eta_i}{\Delta p_i}}$ characteristic bubble radius, m
$R_{cr} = 2\sigma/\Delta p_i$ critical bubble radius, m
$R_g = \Re/\mu$ reduced gas constant, J/(kg·K)
$R_*$ characteristic bubble radius at the beginning of diffusion stage, m
$S$ cell radius, m
$t$ time, s
$t_0$ characteristic time of the process, s
$t_0^D = R_0^2/D$ characteristic time of diffusion process, s
$t_0^\eta = \eta_i/\Delta p_i$ characteristic time of pressure relaxation, s
$t_*$ characteristic time of the beginning of diffusion stage, s
$T$ temperature, K
$v_r = \dot{R}R^2/r^2$ radial velocity of liquid, m/s

*Greek symbols*
$\alpha$ degree in Henry's law
$\beta$ coefficient
$\gamma = \Delta p_i/p_i$ nondimensional value of decompression
$\delta = R_i/R_{cr} - 1$ parameter
$\varepsilon = \rho_l \Delta C_i/\rho_{gf}$ supersaturation of liquid (nondimensional criterion of phase change)
$\zeta$ nondimensional integration variable
$\eta$ dynamic viscosity, Pa·s
$\hat{\eta}$ "effective" dynamic viscosity, Pa·s
$\eta^*$ empirical coefficient, Pa·s
$\kappa = E^*_\eta k_\eta \Delta C_i/(k_B T)$ nondimensional coefficient
$\mu$ molar mass of gas, kg/mole
$\rho$ density, kg/m$^3$
$\sigma$ surface tension, J/m$^2$
$\tau = Dt/R_0^2$ nondimensional time (Fourier number)
$\chi = \bar{r}/\overline{R}$ nondimensional variable

*Gothic symbols*
$\Re$ universal gas constant, J/(mole·K)

*Subscripts*
$i$ initial state
$f$ final state
$g$ gas phase
$l$ liquid phase
"0" characteristic value

applications to different physical processes (degassing, boiling, etc.), can be found in monographs [20,21]. The details of the equation governing the bubble dynamics are still under investigation. In particular, experimental and theoretical modeling of diffusive growth of gas bubbles is described in [22–26], while the nucleation stage in the process of degassing is discussed [27–29].

Following the important contribution by Sparks [30], geophysicists became interested with kinetics the gas-bubble growth in magma melts, that evolved into a separate research field. Indeed, the kinetics is significantly enriched by unique physicochemical properties of magma melts [31], which are characterized by high concentration of dissolved volatile components (mostly water, with mass concentration reaching several percent, depending on saturation pressure), and by high viscosity (which varies in a very wide range: from $10^2$ to $10^{12}$ Pa·s, depending on dissolved gas concentration and the portion of crystallites). This imposes particular restrictions on the domain of applicability of some theoretical models (generally, the equilibrium ones), and requires further research.

While this paper does not purport to be an exhaustive review, let us recall some interesting and significant contributions related to the problem under consideration. The kinetics of nucleation and gas-bubble growth in magma under decompression has been investigated in [32–35]. Theoretical modeling of diffusive bubble growth in magma melts (within the framework of a steady-state model), supported by a set of experimental studies, is described in [36–38]. An original research on bubble growth dynamics is given in [39–41]. The detailed numerical investigations of the bubble growth problem, with a proper account for accompanying thermodynamic processes, are presented by [42–44].

There is yet no model (or solution), which would accurately describe the bubble growth in magma melt within the whole range of governing parameters (including the role of magma supersaturation after decompression), at all stages of the process, and under highly nonequilibrium conditions. The importance of the latter factor is conditioned by the high viscosity of magma and possible nonstationary external forcing. The exact solution obtained in [27] takes into account all of the nuances related to phase transitions at highly nonequilibrium conditions. However, it can only be applied to the diffusive stage of the bubble growth and in the case of instantaneous decompression. In spirit of [27], similar solutions describing the process of the supercooled melt crystallization were obtained in [45,46]. Following the ideas developed in these works, we shall try to describe the process of the bubble growth in magma melts at all stages, including the dynamical one, and the case of high deviations from equilibrium.

## 2. Statement of the problem

Let us consider a highly-viscous gas-saturated liquid (magma melt), which undergoes rapid decompression. We distinguish two types of decompression: (i) instantaneous one, when liquid pressure drops from initial $p_i$ to final $p_f$ value during an infinitesimal period of time, and (ii) gradual one, when the rate of decompression is finite. It is obvious that in the case of a finite decompression rate all the parameters characterizing the state of the system are functions of time, whereas the final value of pressure may be uncertain (e. g. due to changing external conditions).