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Lattice Boltzmann simulation of viscoplastic fluids on natural convection in inclined enclosure with inner cold circular/elliptical cylinders (Part II: Two cylinders)

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ABSTRACT

In this paper, natural convection in an inclined heated cavity with two inner cold circular/elliptical cylinders filled with viscoplastic fluids has been simulated by Lattice Boltzmann Method (LBM). In this study, the Bingham model without any regularization has been studied and moreover viscous dissipation effect has been analysed. Fluid flow, heat transfer, and yielded/unyielded parts have been conducted for certain pertinent parameters of Rayleigh number ($Ra = 10^4, 10^5$ and 10^6), Eckert number, the size of the inner cylinder, various inclined angles of the cavity ($\theta = 0°, 40°, 80°, 120°$), the ratio of the inner cylinder radii (A = 0.25, 0.5, 1, 2, and 4), and different positions of the inner cylinder. Moreover, the Bingham number (*Bn*) is studied in a wide range of different studied parameters. Results indicate that the enhancement of the Rayleigh number augments the heat transfer, with a decrease in the size of the unyielded zones. For specific Rayleigh number and the position of the cylinder, the increase in the Bingham number declines the heat transfer and expands the unyielded sections between the inner cylinders and the enclosure. The rise of the cylinder size in the enclosure enhance heat transfer and alters the unyielded parts. The enhancement of the ratio of the inner cylinder radii augments the heat transfer and declines the unyielded sections. The increases in the vertical distance between two centers of the cylinders enhances heat transfer and moreover, alters the size and shape of the unyielded zones. The increase in the inclined angle of the enclosure alters the heat transfer and the yielded/unyielded zones noticeably. The rise of Eckert number even for higher range of practical values ($Ec = 0.01$, 0.1, and 1) alters the heat transfer and unyielded parts marginally, so the viscous dissipation term can be negligible in this study.

1. Introduction

Natural convection of viscoplastic fluids in an enclosure due to its wide applications and interest in various chemical, metal, and food industries has been considered recently by researchers. Vola et al. [1] studied the natural convection in a cavity filled with a viscoplastic fluid using the Bingham model without any regularisation of the constitutive law. They applied a numerical method based on the combination of the characteristic/Galerkin method to cope with convection and of the Fortin-Glowinski decomposition/coordination method to deal with the non-differentiable and nonlinear terms that derive from the constitutive law. However, the streamlines and isotherms for various yield stress values were limited to one value of the Rayleigh number ($Ra = 10^4$). Turan et al. [2] conducted a study into the simulations of natural convection in square enclosures filled with an incompressible Bingham fluid. The considered flow was laminar and steady. The commercial package FLUENT was utilized to solve the problem. In this study, a second-order central differencing scheme was used for the diffusive terms and a second order up-wind scheme for the convective terms. Coupling of the pressure and velocity fields was achieved using the SIMPLE algorithm. It should be noted that the default Bingham model in FLUENT is a bi viscosity model. The heat transfer and the flow velocities were investigated over a wide range of Rayleigh and Prandtl numbers. They found that the average Nusselt number augments with the rise of the Rayleigh number for both Newtonian and Bingham fluids, whereas the Nusselt numbers of Bingham fluids were smaller than those in Newtonian fluids for a fixed nominal Rayleigh number. They also mentioned that the mean Nusselt number of Bingham fluids decreased with an increase in the Bingham number. Moreover, it was observed that the conduction dominated regime occurs at large values of Bingham numbers. Finally,

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Nomenclature

A	the first Rivlin-Ericksen tensor
A	the ratio of the inner cylinder radii
a	horizontal radius
Bn	Bingham number
b	vertical radius
c	lattice speed
c_p	specific heat capacity at constant pressure
Ec	Eckert number
F	external forces
f_α	density distribution functions for the specific node of α
f_α^{eq}	equilibrium density distribution functions for the specific node of α
g_α	internal energy distribution functions for the specific node of α
g_α^{eq}	equilibrium internal energy distribution functions for the specific node of α
g	gravity
k	thermal conductivity
L	length of the cavity
N	body force
Nu	Nusselt number
p	pressure
Pr	Prandtl number
Ra	Rayleigh number
T	temperature
t	time
x, y	cartesian coordinates
x_{c1}, y_{c1}	the horizontal and vertical positions of the first cylinder center
x_{c2}, y_{c2}	the horizontal and vertical positions of the second cylinder center
u	velocity in x direction
U	the buoyancy velocity scale
v	velocity in y direction

Greek letters

β_T	thermal expansion coefficient
ϕ	relaxation time
τ	shear stress
τ_y	yield stress
ξ	discrete particle speeds
Δx	lattice spacing in x direction
Δy	lattice spacing in y direction
Δt	time increment
α	thermal diffusivity
ρ	density of fluid
ψ	stream function value
$\mathbf{\Lambda}$	the viscoplasticity constraint
θ	the inclined angle of the cavity
$\mathbf{\Omega}$	the vertical distance between the centers of the two cylinders

Subscripts

avg	average
B	bottom
C	cold
$c1$	center of the first cylinder
$c2$	center of the second cylinder
d	dynamic
H	hot
L	left
x, y	cartesian coordinates
α	specific node
R	right
s	static
T	top
tot	total

they reported that for low Bingham numbers, the mean Nusselt number increases with the enhancement of the Prandtl number; by contrast, the opposite behavior was observed for large values of Bingham numbers. Turan et al. [3] continued their studies with analysing the effect of different aspect ratios (the ratio of the height to the length) of the cavity, adding to their previous results that the average Nusselt number follows a non-monotonic pattern with the aspect ratio for specific values of the Rayleigh and Prandtl numbers for both Newtonian and Bingham fluids. At small aspect ratios, the conduction is dominant whereas convection remains predominantly responsible for the heat transfer for large values of aspect ratios. In addition, it was found that the conduction dominated regime occurred at higher values of the Bingham numbers for increasing values of the aspect ratio for a given value of the Rayleigh number. Turan et al. [4] scrutinised the laminar Rayleigh-Bnard convection of yield stress fluids in a square enclosure. The applied method and the achieved results were similar to the two previous studies. Huilgol and Kefayati [5] studied natural convection in a square cavity with differentially heated vertical sides and filled with a Bingham fluid without any regularisation. The finite element method (FEM) based on the operator splitting method was utilized to solve the problem. It was observed that for specific Rayleigh and Prandtl numbers, the increase in the Bingham number decreases the heat transfer. Furthermore, it was found that the growth of the Bingham number expands the unyielded sections in the cavity. Finally, they mentioned that for fixed Rayleigh and Bingham numbers, the unyielded regions grow with the augmentation of the Prandtl number. Karimfazli et al. [6] explored the feasibility of a novel method for the regulation of heat transfer across a cavity. They used computational simulations to resolve the Navier-Stokes and energy equations for different yield stresses.

Many studies have conducted the effect of the presence of the body inside the enclosure on the natural convection of Newtonian fluids and focused on the diverse body shapes such as circular, square and triangular cylinders [7–14]. Just recently, natural convection of viscoplastic fluids is investigated, using regularized models inside cavities with hot and cold bodies. Baranwal and Chhabra [15] studied laminar natural convection heat transfer to Bingham plastic fluids from two differentially heated isothermal cylinders confined in a square enclosure. They utilized regularization approaches of biviscosity and the Bercovier and Engelman models. They used the finite element method-based solver, COMSOL Multiphysics (version 4.3a) to solve the governing equations. Dutta et al. [16] investigated the effects of tilt angle and fluid yield stress on the laminar natural convection from an isothermal square bar cylinder in a Bingham plastic fluid confined in a square duct. They also applied the same regularization approaches of biviscosity and the Bercovier and Engelman models. They also applied the finite element method-based solver, COMSOL Multiphysics (version 4.3a) to solve the governing equations.

Lattice Boltzmann method (LBM) has been demonstrated to be a very effective mesoscopic numerical method to model a broad variety of complex fluid flow phenomena [17–28]. Lattice Boltzmann