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Heat source layout optimization for two-dimensional heat conduction using iterative reweighted L1-norm convex minimization

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ABSTRACT

Optimization of heat source distribution in two dimensional heat conduction for electronic cooling problem is considered. Convex optimization is applied to this problem for the first time by reformulating the objective function and the non-convex constraints. Mathematical analysis is performed to describe the heat source equation and the combinatorial optimization problem. A sparsity based convex optimization technique is used to solve the problem approximately. The performance of the algorithm is tested by several cases with various boundary conditions and the results are compared with a uniformly distributed layout. These results indicate that through proper selection of the number of grid cells for placing the heat sources and a minimum inter-source spacing, the maximum temperature and temperature non-uniformity in the domain can be significantly reduced. To further assess the capabilities of the method, comparisons to the results available in the literature are also performed. Compared to the existing heat source layout optimization methods, the proposed algorithm can be implemented more easily using available convex programming tools and reduces the number of input control parameters and thus computation time and resources while achieving a similar or better cooling performance.

1. Introduction

During operation of all electronic devices and circuits, certain amount of excess heat is generated. The maximum temperature of the electronics and the temperature uniformity of the domain are important criteria that affect the operation consistency and life of the devices. In order to improve reliability of device operation, unify electronic circuit aging processes and reduce electronics failure probability, thermal management of the overall system is required to be studied. Thermal management deals with dissipating the generated heat efficiently and reducing the maximum temperature of the electronics while approaching a uniform temperature distribution in the domain. The need for thermal management is even more crucial with the fast development of today's electronic production and component integration technology in which the size of the electronics has become smaller and the power density has increased noticeably.

One effective way to enhance the heat conduction performance of the system and reduce the maximum temperature of the domain is to insert highly thermal conductive materials such as diamond or carbon fiber [1,2] that are able to reduce the local thermal resistance. In this case, the distribution of such materials is to be optimized with the aim of minimizing the maximum temperature. A variety of approaches have been discussed in the literature including the constructal theory [3–5], entransy theory [6–8], bionic optimization [9,10] and combinatorial algorithms [11,12]. The results of these algorithms have shown that the optimized conduction path formed by the inserted materials has a shape similar to a tree with several branches varying in number and size.

Although conduction cooling employing high thermal conductivity materials has been shown to be an effective method, the optimal distribution of such materials are difficult to realize in practice and the design costs may increase. An alternative approach is to provide passive cooling via surface heat emission or convection and by optimizing only the positions of the electronics (or the heat sources).

For layout optimization problems with large numbers of degrees of freedom, combinatorial algorithms such as genetic algorithm with artificial neural network [13–16] and particle swarm optimization [17] have been used in the literature. In their recent studies, Chen et al. applied bionic optimization [18,19] and simulated annealing [20] methods to find out the optimal source distribution for varying number (up to several tens) of heat sources, which provided significant reduction in the maximum tempera-

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Nomenclature

Δ	thickness of the domain and the heat sources in the 3D model, m
δ	domain extension length for the heat sink realization in the 3D model, m
ϵ	parameter for the algorithm stability, 1
Γ	area of a single heat source, m^2
ϕ	heat source distribution function, W/m^2
Φ_0	volume heat density of a single heat source in the 3D model, $W/(m^3{\cdot}K)$
ϕ_0	intensity of a single heat source, W/m^2
ϕ_b	background heat source distribution before adding the heat sources, W/m^2
σ	standard deviation of the temperature field in the domain, 1
σ_m	normalized standard deviation of the temperature field in the domain, 1
$\mathbf{w}$	vector ($N_g \times 1$) of selection weights (within [0,1]) of grid cells, 1
ε	surface emissivity coefficient, 1
d_g	guaranteed minimum inter-source spacing defined at the algorithm input, m
$d_{\min}$	resulting minimum inter-source spacing at the algorithm output, m
h	convective heat transfer coefficient, $W/(m^2{\cdot}K)$
k	thermal conductivity, $W/(m{\cdot}K)$
L	side length of the square domain, m
l	side length of a square-shaped heat source, m
N_g	number of grid cells for placing the heat sources, 1
N_s	number of heat sources placed in the domain, 1
N_T	number of temperature field samples in the domain, 1
R_m	normalized maximum temperature rise, 1
S	separation matrix ($N_g \times N_g$) defining the minimum inter-source spacing, 1
T	temperature field, K
T_0	environmental temperature, K
T_b	background temperature field before adding the heat sources, K
T_i	temperature rise due to adding the ith source, K
t_T	computation time for the resulting temperature field after the optimization, s
T_{avg}	average temperature of the domain, K
$T_{\max}$	maximum temperature of the domain, K
t_{opt}	total computation time for the optimization with the minimum required number of iterations, s
T_{ref}	reference temperature field with uniform heat generation in the domain, K
t_{ref}	computation time for the reference temperature field calculation, s
w_j	selection weight of the source on the jth grid cell, 1
x, y	horizontal and vertical coordinates of the domain, m
$Z^{(j)}$	diagonal weighting matrix ($N_g \times N_g$) in the jth iteration of the optimization

ture of the domain when compared to the randomly or uniformly distributed heat sources.

In this paper, the layout optimization is addressed using a new method that has not been investigated in the literature. The solution algorithm is based on the sequential reweighted l_1-norm minimization technique. This technique was first introduced by Candes et al. [21] with the aim of minimizing a convex measure of the l_0-norm which is equal to the cardinality of the sources. Recently, the sequential convex optimization method has been effectively used in the domain of sparse antenna array synthesis [22–24] in order to optimize the positions and excitations of the antenna elements. Considering the amplifiers powering the antennas as heat sources, a direct analogy can be made with the heat conduction problem.

Motivated by this analogy, the mathematical modeling of the two-dimensional heat conduction problem is performed and the positions of the heat sources are optimized using the convex optimization algorithm iteratively. The performance is tested using three typical cases with various boundary conditions that were previously applied by Chen et al. [18–20] and an additional case which takes into account the heat emission from the domain surface by convection or radiation while solving the conduction problem.

The remaining parts of the paper are organized as follows. Section 2 presents the mathematical modeling of the two-dimensional heat conduction problem. Section 3 explains the formulation of the layout optimization problem via sequential convex optimizations. Section 4 shows and discusses the results of the four test cases. Section 5 presents the conclusions.

2. Mathematical modeling of the problem

In this section, the mathematical model for the heat conduction optimization problem in a two-dimensional rectangular domain is revised. As previously stated by Chen et al. [18–20], the temperature field (T) satisfies the following equation at steady state

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\phi(x,y)=0$$
$$\text{Boundary}: \quad T=T_0 \quad or \quad k\frac{\partial T}{\partial \mathbf{n}}=0 \quad or \quad k\frac{\partial T}{\partial \mathbf{n}}=h(T-T_0) \tag{1}$$

where k is the thermal conductivity of the domain, ϕ is the heat source distribution function and T_0 is the temperature value at the isothermal boundary or the fluid temperature at the convective boundary. h represents the convective heat transfer coefficient. In fact, Eq. (1) represents a Poisson problem with Dirichlet (isothermal), Neumann (adiabatic) or Robin (convective) boundary conditions that can be solved with a MATLAB-based finite-difference solver [25].

The heat sources in this study are modeled similar to [18] as follows

$$\phi(x,y)=\begin{cases}\phi_0, & (x,y)\in\Gamma\\ 0, & (x,y)\notin\Gamma\end{cases} \tag{2}$$

where ϕ_0 is the (constant) intensity of a single heat source and Γ represents the area of that heat source.

If a background temperature T_b and a source distribution ϕ_b are assumed, Eq. (1) can be expressed as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T_b}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T_b}{\partial y}\right)+\phi_b(x,y)=0$$
$$\text{Boundary}: \quad T_b=T_0 \quad or \quad k\frac{\partial T_b}{\partial \mathbf{n}}=0 \quad or \quad k\frac{\partial T_b}{\partial \mathbf{n}}=h(T_b-T_0) \tag{3}$$

When the ith source is added into the domain, the temperature rise T_i is given by

$$\frac{\partial}{\partial x}\left(k\frac{\partial T_i}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T_i}{\partial y}\right)+\phi_i(x,y)=0$$
$$\text{Boundary}: \quad T_i=0 \quad or \quad k\frac{\partial T_i}{\partial \mathbf{n}}=0 \quad or \quad k\frac{\partial T_i}{\partial \mathbf{n}}=hT_i \tag{4}$$