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Internal heat transfer coefficient estimation in three-dimensional ducts through the reciprocity functional approach – An analytical approach and validation with experimental data

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In the current paper it is presented and tested an innovative method for estimating internal heat transfer coefficients in ducts adopting only thermal measurements acquired on the external wall surface. This approach is based on the reciprocity functional analysis, which is a powerful non-intrusive inverse problem technique. The adoption of this technique is promising since it avoids intrusive measurements, it is fully non-iterative and the computational time and cost are very limited. In this paper, it is presented an extension of the classical reciprocity functional methodology in the sense that a fully analytical expression, obtained by the integral transform technique, is developed for estimating the internal heat transfer coefficient in a three-dimensional problem. Such methodology, avoiding the solution of linear systems, reduces the computational costs that are massive when the traditional approaches are applied to three dimensional problems. The proposed procedure is first validated adopting synthetic temperature data and then tested using real temperature measurements acquired by an infrared camera. The results highlight that the methodology is able to recover the unknown functions in a very short computational time with a good accuracy.

1. Introduction

Convection processes are very important in many engineering application, such as thermal power plants, heat exchangers, oil pipelines, and food pasteurization systems, among others. In several of these applications, it is very important to know the spatial distribution of the heat transfer coefficient on the interior surface of some ducts. However, it is very difficult to directly measure such local heat transfer coefficient and correlations are usually rare and unreliable. A straightforward solution for this interesting issue can be found by applying an Inverse Heat Conduction Problem (IHCP) technique. This approach enables to estimate the local heat transfer coefficient on the internal surface of the duct by monitoring only the external wall surface temperature distribution.

IHCPs present some complications because they are ill-posed, which implies that they are significantly sensitive to fluctuations in the input data, such as those caused by experimental noise. To bypass these difficulties, which are particularly critical when infrared thermography is used as the temperature measuring technique, many methods based on experimental data processing have been suggested and validated in the literature. Among these methods we can cite gradient based methods [1], methods based on Green's functions [2], regularization techniques [3], mollification algorithms [4], and Bayesian techniques [5,6].

Among the numerous possible applications of this approach, the estimation of the temperature, the heat flux, or the convective heat transfer coefficient distributions on the internal wall of a duct, using only the external wall temperature distribution, is very appealing. This application is particularly attractive because it can be employed both in the industrial applications to monitor productive processes and in the research field to investigate the heat transfer mechanisms.

In [7,8] the conjugate gradient method was used to solve an IHCP to estimate the transient temperature profile near the inner

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Nomenclature

Abbreviations

CITT	Classical Integral Transform Technique
IHCP	Inverse Heat Conduction Problem
RF	Reciprocity Functional
TRN	Thermal Resistance Network
SVD	Singular Value Decomposition

Symbol

E_h	estimation error
f	orthonormal basis function
g	heat source per unit volume [W/m^3]
h	convective heat transfer coefficient [W/m^2·K]
K	thermal conductivity or Bessel third order function [W/mK or –]
L	tube length [m]
M, N	integers related to the number of basis function
q	heat flux per unit area [W/m^2]
r	radial coordinate [m]
R	residues [K]
Re	Reynolds number
S_0, S_1	internal and external surface [m^2]
T	temperature [K]
x, y	Cartesian coordinates [m]
Y	measured temperature [K]
z	axial coordinate [m]
α	linear expansion coefficient related to the first auxiliary problem
β	linear expansion coefficient related to the second auxiliary problem
μ	generic basis function in the first auxiliary problem
ξ	generic basis function in the second auxiliary problem
σ	standard deviation of the temperature measurements [K]
ε	random variable with Gaussian distribution
λ	eigenvalues of an associated eigenvalue problem
Γ_0, Γ_1	lateral surfaces
Ω	domain of the physical problem
Φ	harmonic test function in first auxiliary problem
Ψ	harmonic test function in second auxiliary problem
θ	angular coordinate [rad]

Subscripts, superscripts

b	bulk
env	environment
ext, 1	external
int, 0	internal
i, j, n, m	integers
p, q, v	integers
$meas$	measurement
opt	optimal

wall of a pipe elbow with thermal stratification. A two-dimensional (2D) analysis was considered and only four temperature measurements points, located on the external wall, were used as input data for this method. The obtained results were good with a maximum deviation less than 1% in each measurement point. Such results were particularly useful in cases where there was no significant temperature gradient along the longitudinal axis of the duct. One of the recommendations presented in [7,8] was to the increase the number of temperature measurement and extend the analysis to three-dimensional cases.

Noh et al. [9] solved a 3D IHCP to estimate a transient heat flux on the internal surface of a two-layer cylindrical duct using the Kalman filter. One layer was made of chromium and the other one of steel. The properties of both materials were considered constants and a perfect thermal contact between the layers was considered. Synthetic external temperature measurements, obtained using a commercial software, were used as input data to the inverse problem solution. However, the Thermal Resistance Network (TRN) method was used to solve the direct problem, as part of the inverse problem technique, in order to reduce the computational time required for the estimate. Several profiles for the transient internal heat fluxes were restored with good results.

Yang et al. [10] formulated a thermo-elastic hyperbolic inverse problem to estimate thermal stresses and heat fluxes on the internal wall of a circular duct. The inverse problem was formulated as an optimization problem, which allowed the use of the conjugate gradient method together with the discrepancy principle. Synthetic temperature measurements, taken on the same inner wall of the duct, were adopted as the input data of the formulated inverse problem. Transient heat fluxes on the inner wall and temperature stresses within the duct were estimated with good results.

The temperature and the heat flux distribution on the inner wall of a micro-channel were estimated by Rouizi et al. [11] acquiring temperature on the external wall. The quadrupole method was employed with good results.

Bozzoli et al. [12] solved an Inverse Heat Conduction Problem using the Tikhovov regularization method to estimate the heat transfer coefficient in a duct adopting a two-dimensional analysis. The 2D analysis was conducted by neglecting the temperature gradient along the duct. Therefore, the direct problem was obtained as the solution of a heat conduction problem in the cross section of the duct, subjected to natural convection on the external wall and a prescribed heat flux on the internal wall. The methodology used to estimate the heat transfer coefficient consisted in firstly estimating the internal heat flux and then using it to obtain the internal heat transfer coefficient. The choice of the regularization parameter was made iteratively, following the methodology developed in [13,14]. The estimates were very good.

An IHCP was used to estimate a spatially distributed heat transfer coefficient on the internal wall of a duct in [15]. The inverse problem approach was based on a filtering method, which removes the unwanted noises from the temperature measurements. In [15], the authors measured the steady state external temperature with a thermographic camera and considered it as the input data for the IHCP in order to estimate the internal heat transfer coefficient. This temperature data was filtered and considered equal to the internal wall temperature due to the validity of Biot hypothesis (i.e. thin wall approximation) in the radial direction. The Biot hypothesis allowed an explicit calculation of the internal heat transfer coefficient, subjected to a suitable filtering of the measured external temperature. Synthetic temperature measurements, with different convection profiles, were also used to estimate the heat transfer coefficient and the results showed a good agreement compared to the exact values.

Colaço et al. [16] estimated the heat transfer coefficient in a duct using the Reciprocity Functional Technique, which is a non-iterative inverse problem technique [17–19]. In [16], a 2-D inverse heat conduction problem with heat generation within the wall thickness was considered. The physical properties were supposed constants and known. The temperature of the fluid in contact with