International Journal of Heat and Mass Transfer 121 (2018) 37–45

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# Nonlinear convection regimes in superposed fluid and porous layers under vertical vibrations: Positive porosity gradients

E.A. Kolchanova [a,b,c,*], N.V. Kolchanov [c]

[a] Institute of Continuous Media Mechanics UB RAS, 1 Academ. Koroleva, Perm 614013, Russian Federation
[b] Perm National Research Polytechnic University, 29 Komsomolsky prospekt, Perm 614990, Russian Federation
[c] Perm State University, 15 Bukireva, Perm 614990, Russian Federation

A R T I C L E I N F O

*Article history:*
Received 10 November 2017
Received in revised form 15 December 2017
Accepted 28 December 2017

*Keywords:*
Nonlinear convection regimes
Inhomogeneous porous medium
Superposed fluid and porous layers
Single-component fluid
High-frequency and small-amplitude vibration

A B S T R A C T

We investigate the onset of average convection and its nonlinear regimes in a single-component fluid layer overlying a fluid-saturated porous layer. A heated from below cavity with a superposed fluid and a porous medium undergoes high-frequency and small-amplitude vertical vibrations in the gravitational field. Porosity of the medium decreases linearly with depth at a positive porosity gradient. Thermal vibrational convection equations are obtained by the averaging method and solved numerically. The shooting method, Galerkin method and finite-difference method are applied. It is shown that for small vibration accelerations, a convective flow is generated as short-wave rolls in the fluid layer overlying a porous medium. The heat flux undergoes abrupt changes as the supercriticality increases. It is due to the fluid flow penetrating into pores. A magnitude of the jump grows with the growth of vibration intensity. For sufficiently large vibration accelerations, the average convection is excited in the form of long-wave rolls that penetrate both layers. Here, the Nusselt number is 2–3 times higher than its value in the static gravity field.



## 1. Introduction

Thermal convection in a differentially heated fluid layer overlying a fluid-saturated porous layer under the gravity field has a number of characteristic features different from the convection in a single-layer system [1–7]. Depending on the parameters of layers (the ratio of layer thicknesses, the ratio of thermal conductivities, the Darcy number, etc.), convection can be excited as the short-wave rolls occurring in the fluid layer overlying a porous medium or as the long-wave rolls spreading through both layers. In this case, neutral stability curves of the fluid equilibrium in the layers are bimodal. They have two minima, corresponding to perturbations with a small and large wave number. In [2], convection was experimentally studied in a layer of aqueous glycerin solution 4 cm thick heated from below and partially filled with glass balls of 3 mm in diameter. The balls divided the layer into two parts, comprising a porous and non-porous medium. The formation of convective structures was observed after the loss of equilibrium stability of the system. An eightfold decrease in their wavelength was obtained for the ratio of the liquid medium layer thickness to that of the solution-saturated porous medium ranging from 0.1 to 0.2.

Unlike heat transfer by conduction, convection that occurs in the layers upon reaching the threshold Rayleigh number $R_{m*}$ causes an increase of the heat flux with the growth of supercriticality $R_m/R_{m*}$. A degree of the heat flux intensification strongly depends on the type of the fluid flow in a two-layer system. For a large relative thickness $d = h_f/h_m \geqslant 0.2$ of the fluid layer overlying a porous medium, convection in the layer is generated as the short-wave rolls. In this case, a heat flux from the lower hot boundary of the porous medium increases quite insignificantly with the growth of supercriticality [3]. For $d < 0.2$, convection is excited as the long-wave rolls penetrating into medium pores. A substantial increase in the Nusselt number with the growth of supercriticality was recorded in [3]. It was also found [4] that characteristic of thick porous layers are the oscillations associated with the formation of additional vortices in the fluid layer overlying the porous medium.

Changes in porosity with depth affect the threshold for equilibrium instability in a two-layer system. With the growth of porosity gradient, the threshold for the onset of long-wave convection changes more essentially than that of the short-wave convection [5,6]. The nonlinear convection regimes in a fluid layer partially

* Corresponding author at: Institute of Continuous Media Mechanics UB RAS, 1 Academ. Koroleva, Perm 614013, Russian Federation.

*E-mail addresses:* kolchanovaea@gmail.com (E.A. Kolchanova), kolchanovn@gmail.com (N.V. Kolchanov).

https://doi.org/10.1016/j.ijheatmasstransfer.2017.12.144
0017-9310/

filled with an inhomogeneous porous medium were studied in [7]. Porosity was linearly dependent on the vertical coordinate. A positive porosity gradient $m_z$ was set in the medium, in which porosity decreases with depth. In this case, short-wave convection was generated at sufficiently large values of $m_z$, and a heat flux through both layers increases weakly with the growth of supercriticality. The study revealed a heat flux jump in the homogeneous medium with $m_z = 0$, which was due to a convective flow penetrating pores at sufficiently high supercriticalities. The heat flux oscillations were observed in the medium, in which porosity increases with depth at $m_z = -0.2$ for $R_m/R_{m*} > 3.4$. They were attributed to the formation of additional vortices in the fluid layer, which interact with the long-wave rolls, penetrating the pores of the medium. The long-wave rolls arose at the threshold Rayleigh number $R_{m*}$.

Convection generated in a fluid layer [8,9] or a fluid-saturated porous layer [10–13] heated from below and oscillating in a vertical direction with high frequency and small amplitude is referred to as the average convection. The threshold for the onset of this convection increases with the growth of vibration acceleration, so that high-frequency vertical vibrations stabilize the mechanical equilibrium of the fluid in the layer. The finite-frequency vibrations can specify not only the synchronous but also the subharmonic response of the fluid to an external periodic action [14–19], which makes possible the excitation of parametric convection. Vibrations destabilize the equilibrium state with respect to subharmonic perturbations. In addition to synchronous and subharmonic oscillations the quasiperiodic instability was observed in a binary fluid layer heated from below in the modulated gravity field [20–22]. It was associated with the growth of the perturbations characterized by two different frequencies: the frequency of natural neutral oscillations and the frequency of the external field.

The effect of high-frequency vertical vibrations on the nonuniformly heated single-component fluid in a layer partially filled with a porous medium in the gravity field was studied in [23–26]. It was shown that the stability threshold for equilibrium state and the wavelength of the most dangerous perturbations increase with the growth of vibration acceleration. The intensification of vibrations leads to an abrupt change of instability. If, in the absence of vibrations convection is initiated in the form of short-wave rolls, then at sufficiently high vibration acceleration the average convection occurs in the form of long-wave rolls penetrating into a porous medium. In [26], the nonlinear regimes of average convection were considered. It was found that for small relative thicknesses of the fluid layer overlying a porous medium, the supercriticality range for the oscillatory convection regime decreases with the growth of the vibrational Rayleigh number. Vibrations also play a stabilizing role. They prevent the formation of additional short-wave vortices at the interface between the layers, which cause oscillations of the fluid velocity when interacting with long-wave vortices. For comparable layer thicknesses, a hysteresis was detected at some vibrational Rayleigh numbers. The effect of finite-frequency vibrations on the fluid in the two-layer system heated from below was investigated in [27]. It was shown that much higher vibration amplitudes are required to obtain the long-wave convection.

A linear stability problem for a single-component fluid layer overlying an inhomogeneous porous layer heated from below in the gravity field under high-frequency vertical vibrations was considered in [28]. Porosity was assumed to be linearly dependent on the vertical coordinate. It was found that a jump in the wavelength of the most dangerous perturbations for the stability threshold was replaced by its smooth change at higher vibration accelerations and positive porosity gradients. The greatest stabilizing effect of vibrations was achieved in a medium whose porosity decreases with depth.

This paper is a continuation of work [28]. Here, we study the nonlinear regimes of average convection in the fluid and porous layers under vertical high-frequency and small-amplitude vibrations. Porosity of an inhomogeneous porous medium decreases linearly with depth at a positive porosity gradient $m_z$. We represent the medium as a system of packed spheres, which do not move relative to each other. The last remark is important because in the system with freely moving spheres under the action of vertical vibrations a hilly relief may occur at the interface between the layers [29]. In experimental work [29], in which glass balls were used, the authors observed a transition to a completely liquefied state with increasing vibration acceleration. In this case, the interface between the layers vanishes.

## 2. Governing equations

Let us explore the onset of convection and nonlinear convective regimes in a single-component fluid layer partially filled with a porous medium in the gravity field. The medium is considered to be non-uniform in the vertical direction and divides the layer into two parts (Fig. 1). The two-layer system formed has impermeable and heat-conducting external boundaries. It is subjected to vertical vibrations of high frequency $\omega$ and small amplitude $a$.

We assume that porosity variation with the vertical coordinate is governed by a linear law: $m(z) = m_i + m_z z/h_m$, where $m_i$ is porosity near the interface between the layers, $m_z$ is the dimensionless porosity gradient. In this paper, we restrict ourselves to the case of positive $m_z$, at which porosity decreases with depth. A porous medium is represented as a system of packed spheres. Therefore, its permeability is determined by the Carman-Kozeny relation [30,31]: $K = \frac{D^2 m^3}{180(1-m)^2}$, where $D$ is the diameter of spheres.

The equations of thermal vibrational convection are written in the reference frame associated with the oscillating cavity [8,10–12,24,26,28]. A porous medium is considered to be non-deformable. Spheres are fixed and do not move relative to each other but oscillate together with the cavity in the vertical direction. The momentum, energy and mass balance equations in the fluid layer are

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\nabla\right)\vec{v} = -\frac{1}{\rho_f}\nabla p_f + \nu_f \Delta \vec{v} + \beta_T T\left(g - a\omega^2 \cos\omega t\right)\vec{\gamma}, \quad (1)$$

$$\frac{\partial T}{\partial t} + \left(\vec{v}\nabla\right)T = \chi_f \Delta T, \quad (2)$$

$$\mathrm{div}\,\vec{v} = 0, \quad (3)$$

and in the porous layer

$$\frac{1}{m(z)}\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho_f}\nabla p_m - \frac{\nu_f}{K(z)}\vec{u} + \beta_T \vartheta\left(g - a\omega^2 \cos\omega t\right)\vec{\gamma}, \quad (4)$$

$$b(z)\frac{\partial \vartheta}{\partial t} + \left(\vec{u}\nabla\right)\vartheta = \mathrm{div}\left(\chi_{eff}(z)\nabla\vartheta\right), \quad (5)$$

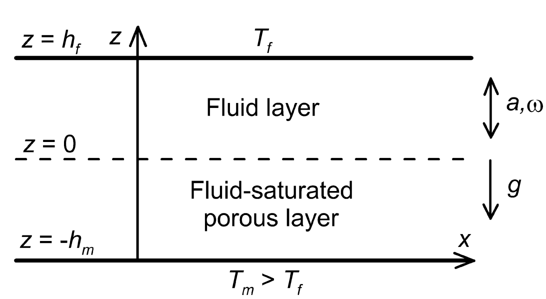


**Fig. 1.** The two-layer system heated from below and subjected to vertical vibration.