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Influence of the computational domain on DNS of turbulent heat transfer up to $Re_\tau = 2000$ for $Pr = 0.71$

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ABSTRACT

We present a new set of direct numerical simulation data of a passive thermal flow in a turbulent plane Poiseuille flow with constant Prandtl number $Pr = 0.71$, and mixed boundary conditions. Simulations were performed at $Re_\tau = 500$, 1000, and 2000 for several computational domains in the range of $l_x = 2\pi h$ to $8\pi h$ and $l_z = \pi h$ to $3\pi h$. As a first key result we found that a length of $l_x = 2\pi h$ and a width of $l_z = \pi$ is enough to accurately obtain the one-point statistics and the budgets of the thermal kinetic energy, its dissipation and the thermal fluxes. None of them collapse exactly in wall units. On the other hand, the value of the thermal Kármán constant grows very slightly with the Reynolds number with a value of $\kappa_{th} = 0.44$ for $Re_\tau = 2000$. The statistics of all simulations can be downloaded from the web page of our http://personales.upv.es/serhocal/group.

1. Introduction

The behavior of turbulent flows is still an open problem in physics and it is probably the one with most implications in day-to-day life. The study of the thermal properties of flows for different values of Reynolds, Prandtl and Rayleigh numbers are becoming more and more important. In a recent study for NASA, Slotnick et al. [1] highlighted the importance of thermal flows in aeronautics applications for the foreseeable future. To cite only another example, for Prandtl number different to the one of air, a better knowledge of the dynamics of thermal flows is needed for the simulation of nuclear Liquid Metal Reactors (LMR) [2,3]. Due to the continuous improvement of computer power, Direct Numerical Simulation (DNS) has become one of the most promising technique to study applications involving drag, noise or heat transmission. Moreover, as experiments involving thermal properties of turbulent wall flows are difficult to perform, DNS appears to be an essential tool.

The first DNS addressing heat transfer in turbulent channels was made by Kim and Moin in 1987 [4], for $Re_\tau = u_\tau h/\nu = 180$ and $Pr = 0.71$. Here, Re_τ is the friction Reynolds number, Pr is the molecular Prandtl number, u_τ is the friction velocity, h is the semi-height of the channel, and ν is the kinematic viscosity. Kim and Moin used an artificial boundary condition to describe the thermal flow, assuming constant heat generation throughout the fluid. This procedure has the advantage that this heat can be treated in the same way that the mean pressure gradient term is in the flow equations. Lyons et al. [5] and Sumitani and Kasagi [6] used a different boundary condition, with walls at different temperature, without considering any effect of buoyancy. Kasagi et al. [7] introduced a more realistic condition in 1992, named mixed boundary condition (MBC from now on). This condition was imposed on the two walls so that the local mean temperature increased linearly in the streamwise direction. With any buoyancy effect neglected, temperature was considered as a passive scalar. Using MBC as boundary condition, the Reynolds number has been increased constantly during the last few years for several Prandtl numbers. Saha et al. [8], performed an excellent review of the different simulations made until very recently. Among these articles, it is worth to mention the work of Abe, Kawamura, and collaborators [9,10]. Up to the knowledge of the authors, the largest DNS was made in 2004 by Abe et al. [10], for $Re_\tau = 1020$ and $Pr = 0.025$ and 0.71. More recently, Duponchel et al. [2] made a Large Eddy Simulation at $Re_\tau = 2000$, that is the largest Re_τ simulated up to now.

One critical point of DNS is the domain to be simulated. Saha et al. in two papers [8,11] investigated the length of the largest turbulent structures in pipes and the comparison between pipes and channels. In fact, in almost all of the papers cited in this introduction and Saha et al.'s works, the size of the computational box is narrower and shorter than in the classical turbulent channel flow simulations. While the more or less classical size of the computational domain for Poiseuille flows is $(8\pi h, 2h, 3\pi h)$ [12–16], in most

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Nomenclature

List of symbols

x_i	coordinate x_i $(\equiv x, y, z)$
y	vertical coordinate normalized with h starting from 0
U_i	velocity in the direction x_i
Θ	temperature
$u_i,\ \theta$	turbulent fluctuations
$\bullet'$	statistic parameter
$\langle\bullet\rangle_{x_i}$	mean value in x_i direction
t	time
$\bullet$	normalized by $h,\ U_b$ and ν
$\bullet^+$	normalized by $u_\tau,\ T_\tau$ and ν
$h,\ \rho$	channel half height and density
$\nu,\ c_p$	viscosity and specific heat at constant pressure
$\kappa_t h$	thermal Kármán constant
u_τ	friction velocity $\left(=\sqrt{\tau_w/\rho}\right)$
T_τ	friction temperature $(=q_w/(\rho c_p u_\tau))$
U_b	bulk velocity $\left(=\langle U_1\rangle_{x,z,t}\right)$
q_w	normal heat flux to the walls
Re	Reynolds number $(=U_b h/\nu)$
Re_τ	Reynolds friction number $(=u_\tau h/\nu)$
Pr	Prandtl number $(=\nu/\alpha)$

of the thermal flow simulations performed until now a box of length $4\pi h$ has been used. It is well known that to accurately represent the largest structures of the flows, longer boxes are needed. Monty et al. [17] found that the length of the largest meandering structures of wall flows is up to $25h$. Moreover, in the case of Couette flows, the length of these structures is even longer, up to at least $80h$, [18] demanding even larger domains.

However, one of the reasons for using short and narrow boxes is that DNS are computationally expensive. This cost can be approximated by $L_x^2 L_y Re_\tau^4 Pr^{3/2}$ [19], limiting the possibilities of running high Reynolds numbers. Lozano-Durán and Jiménez, [20] studying isothermal channel flows found that one-point statistics can be accurately computed in boxes with streamwise and spanwise sizes of only $2\pi h \times \pi h$. In this paper we show that one-point statistic of the thermal flow, including dissipation and kinetic energy budgets, can be accurately represented in this box.

The paper is structured as follows. The numerical method and the validation strategy is explained in the second section. The statistics of the flow, including the turbulent budgets, are discussed in the third section. The turbulent structures of the flow are discussed in the fourth one. The fifth section contains the conclusions.

2. Model description and formulation

In this work, a new set of DNS of a passive thermal flow in a Poiseuille turbulent channel has been conducted within a computational box of $L_x = \alpha\pi h,\ L_y = 2h$ and $L_z = \beta\pi h$, with spanwise and streamwise periodicity, and for several values of α and β (see Fig. 1).

The streamwise, wall-normal, and spanwise coordinates are x, y, and z and the corresponding velocity components are U, V and W or, using index notation, U_i. The temperature is denoted by T and the transformed temperature (see below) by Θ. Statistically averaged quantities are denoted by an overbar, whereas fluctuating quantities are denoted by lowercase letters, i. e., $U = \overline{U} + u$.

The flow can be described by means of the momentum and mass balance equations, and the energy conservation principle. The fundamental equations for the fluid flow are the classical Navier-Stokes flows [21].

$$\partial_t U_i^+ + U_j^+ \partial_j U_i^+ = -\partial_j P^+ + \frac{1}{Re_\tau}\partial_{jj} U_i^+, \tag{1}$$

$$\partial_j U_j^+ = 0. \tag{2}$$

In these equations the superscript (+) indicates that the quantities have been normalized by u_{tau}. As the thermal flow is modeled by the MBC, the averaged temperature grows linearly with respect to x. A transformed temperature Θ is introduced as $\Theta = T - T_w$ to satisfy the constant heat flux boundary condition [9,21]. The great advantage of this method is that Θ is periodic in the streamwise direction and Fourier methods can be applied. The equation for Θ becomes [11],

$$\partial_t \Theta^+ + U_j^+ \partial_j \Theta^+ = \frac{1}{Re_\tau Pr}\partial_{jj}\Theta^+ + U_1^+ / \langle U^+ \rangle_{xyz}. \tag{3}$$

Here, the temperature has been adimensionalised by the friction temperature $T_\tau = q_w/\rho c_p u_\tau$. Using this model, the boundary conditions for the modified temperature are simply $\Theta(y = 0, 2h) = 0$.

Table 1 summarizes the parameters of the present simulations. The wall-normal grid spacing is adjusted to keep the resolution at $\Delta y = 1.5\eta$, i. e., approximately constant in terms of the local isotropic Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$ for every Re_τ case as in [14,22]. In wall units, Δy^+ varies from 0.42 at the wall up to $\Delta y^+ \simeq 7.2$ at the centerline. At the center of the channel the resolutions along the three coordinates are approximately equal, $\eta - 1.8\eta$, [23]. The wall-parallel resolution is the one employed by Hoyas and Jiménez [14] and similar to [16]. As $Pr = 0.7$, the thermal scales are expected to be of the same size that the ones of the flow, so the thermal flow does not need a grid refinement. The mesh sizes given in Table 1 are comparable to the ones of Abe et al. [10] for the streamwise direction and slightly better for the spanwise, as Abe et al. use $\Delta_z^+ = 4.25$ for their $Re_\tau = 1000$ case. Only the case P22 as a larger resolution, but without any effect on the convergence or the statistics (not shown). Cases P5 and P1 has been already computed, [10] but they have been computed again to validate the method used in this work, and to compare the different domains using a single method. Case P21 and P22 are completely new.

Eq. (3) was implemented in LISO code, which has successfully been employed to run some of the largest simulations of turbulence [14,24,18,22]. The Navier-Stokes equations are transformed into an equation for the wall-normal vorticity ω_y and for the Laplacian of the wall-normal velocity $\phi = \nabla^2 v$ as in [12]. The spatial discretization uses dealiased Fourier expansions in x and z direction, and seven-point compact finite differences in y direction with fourth-order consistency and extended spectral-like resolution [25]. The temporal discretization is a third-order semi-implicit Runge-Kutta scheme [26]. In every simulation, the flow had to evolve from an initial file, which has been taken from previous different simulations. The code was run until some transition phase has passed and the flow had adjusted to the new set of parameters. Once the flow is in a statistically steady state, statistics are compiled. The running times used to compile statistics are shown in the rightmost column of Table 1. They are given in terms of wash-outs periods, where U_b is the bulk velocity. The transitions until the simulations reached a statistically steady state, which can be very time consuming, are not contemplated in this table.