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On the stability of thermocapillary convection of a Bingham fluid in an infinite liquid layer

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ABSTRACT

The linear stability of thermocapillary liquid layer of a Bingham-plastic fluid is studied. Due to the yield stress of Bingham fluid, there is a plug region in the flow, which divides the yielded flow into two regions. When the flow is subjected to a small perturbation, the velocity perturbation below the upper surface of plug region is negligible, while the temperature perturbation can be found in all flow regions at moderate and small Prandtl numbers (*Pr*). The perturbation amplitude of the upper surface of plug region decreases rapidly with the increase of *Pr*. The preferred modes are the upstream oblique wave and the downstream streamwise wave at small and large *Pr*, respectively. The effects of the yield stress, gravity and the interfacial heat transfer on the flow stability are discussed. The perturbation amplitude only appears above the plug region, which differs from the cases in the plane Bingham–Poiseuille flow and the thermocapillary liquid layer of a Carreau fluid.

1. Introduction

A liquid layer will be set in motion by a temperature-induced surface tension gradient when a horizontal temperature gradient is imposed on its surface. This flow is called the thermocapillary convection. Due to its important role in crystal growth [1], the thermocapillary convection has been studied extensively [2]. Recently, the thermocapillary flows of non-Newtonian fluids have also received much attention for its great practical importance in film coating [3], film drying [4], dewetting [5], lithography [6], inkjet printing [7] and polymer processing in microgravity [8]. The non-Newtonian effect makes the flow property vary considerably from that of a Newtonian fluid.

The viscoelastic thermocapillary liquid layers have been investigated by many authors [9–12]. It is found that although the elasticity does not change the velocity and temperature distributions in the basic flow, due to the normal stress, the flow stability is affected by the elasticity significantly. There are also a few papers devoted to the study of thermocapillary flows of shear-thinning fluids [13–15]. For linear flow, the shear-thinning effect does not change the basic flow, however, it is destabilizing at small and moderate *Pr* but increases the stability slightly at large *Pr*. For return flow, the shear-thinning effect leads to a viscosity stratification in the basic flow, the streamwise wave is excited at large *Pr*, and a new mechanism is found at moderate *Pr*, where the hot spots appear at the bottom of the layer [15]. However, to the best of our knowledge, the thermocapillary liquid layer of viscoplastic fluids has not been investigated.

Viscoplastic fluids appear in many industrial applications and nature environment, such as drilling muds [16], polymers [17], mucus [18] and lava [19]. The main feature of a viscoplastic fluid is its yield stress: it exhibits liquid-like behaviour when it is sufficiently stressed, and solid-like behaviour when the stress is low. Due to the special property and wide applications, there are many works devoted to the study of viscoplastic fluids. The recent developments have been reviewed by Balmforth, Frigaard & Ovarlez [18].

One of the ideal models for viscoplastic fluids is the Bingham fluid, which exhibits a yield stress and a plastic viscosity [20]. The Bingham fluid has been widely used in theoretical studies for its simplicity [21]. In many Bingham fluid flows, there can be a region where the shear stress is less than the yield stress. It behaves as a rigid body and is called the plug region or unyielded region. The inclusion of a plug region makes the flow stability of a Bingham fluid quite different from those of other fluids.

The shear flow stabilities of Bingham fluids have been studied in many works, which indicate that the flow is stabilized by the effect of the yield stress. Frigaard, Howison & Sobey [16] have examined the stability of plane Poiseuille flow of a Bingham fluid, and found

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Nomenclature

$\hat{a}$	thermal expansion coefficient
b	temperature gradient on the surface
$B = \tau_0 d/\mu_0 \hat{U}_0$	Bingham number
$Bi = \hat{h}d/\hat{k}$	Biot number
$Bo = \rho g \hat{a} d^2/\gamma$	dynamic Bond number
$c = -\sigma_i/k$	wave speed
d	depth of the layer
g	gravitational acceleration
h_0	length of the plug region
$h^{\pm}$	perturbations of the yield surface
$\hat{h}$	surface heat transfer coefficient
$k = \sqrt{\alpha^2 + \beta^2}$	wave number of wave propagation
$\hat{k}$	thermal conductivity
$Ma = b\gamma d^2/\mu_0 \chi$	Marangoni number
$Pr = \mu_0/\rho\chi$	Prandtl number
$\tilde{Q}$	imposed heat flux to the environment
$R = \rho \hat{U}_0 d/\mu_0$	Reynolds number
$(\mathbf{u}, T, P, \boldsymbol{\tau})$	velocity, pressure, temperature and stress
$\hat{U}_0 = b\gamma d/\mu_0$	characteristic velocity
z_0	length of the yielded region
α, β	wave number in the x and y directions
γ	negative rate of change of surface tension with temperature
$\dot{\gamma} = \sqrt{\dot{\gamma}_{ij}\dot{\gamma}_{ij}/2}$	second invariant of $\dot{\boldsymbol{\gamma}}$
$\dot{\boldsymbol{\gamma}}$	strain-rate tensor
$\delta/\delta t$	upper convected derivative
μ	dimensionless effective viscosity
μ_0	plastic viscosity
ρ	fluid density
$\tilde{\sigma}$	surface tension
σ_r, σ_i	growth rate and frequency of small perturbation
$\tau = \sqrt{\dot{\tau}_{ij}\dot{\tau}_{ij}/2}$	second invariant of $\boldsymbol{\tau}$
τ_0	yield stress
ϕ	propagation angle
χ	thermal diffusivity

that the critical Reynolds number increases almost linearly with increasing Bingham number. The nonlinear stability analysis has been performed by Nouar & Frigaard [22]. The results showed that the critical Reynolds number R increases like $R = O(B^{1/2})$ when the Bingham number $B \to \infty$. On the other hand, the three-dimensional linear stability analysis performed by Frigaard and Nouar [23] suggested that when $B \to \infty$, a critical Reynolds number $R = O(B^{3/4})$ is bounded for all wavelengths. The receptivity problem of plane Bingham–Poiseuille flow with respect to weak perturbations has been investigated using modal and non-modal approaches by Nouar et al. [24]. It has been reported that when $B \ll 1$, the optimal disturbance consists of almost streamwise vortices, whereas at moderate or large B, the optimal disturbance becomes oblique. Nouar & Bottaro [25] have revisited the problem for the case in which the idealized base flow is slightly perturbed. The results suggested that very weak defects are indeed capable to excite exponentially amplified streamwise travelling waves. The study of the stability of Bingham fluid flows has been extended to the spiral Couette flow [26] and Taylor-Couette flow [27].

The purpose of this paper is to examine the thermocapillary convection of a Bingham fluid in an infinite liquid layer and its stability, which have not been studied before. The works of Bingham fluid have demonstrated that the flow stability depends on the Bingham number obviously. So we have reasons to believe that there can be something new in the thermocapillary convection of a Bingham fluid, which are different from those in other fluids.

The paper is organized as follows. In Section 2, the physical model and numerical descriptions of the problem are presented. The basic flow solutions and perturbation equations are derived. Then in Section 3, critical parameters at different Bingham number, Bond number and Biot number are obtained; the perturbations of the velocity, temperature and yield surface are displayed and the energy mechanism is studied; the instability is discussed and comparisons are made with other fluids and flows. Finally, our conclusions are presented in Section 4.

2. Problem formulation

The model of thermocapillary liquid layer [28] is applied in the present work, where a fluid layer on an infinite rigid plane is subjected to a temperature gradient on the free surface. The instability behaviours predicted by this model have been observed in both experiment [29] and numerical simulation [30]. In Fig. 1, d is the depth of the layer, U_0 is the velocity, x, y, z are the streamwise, spanwise and wall-normal directions, respectively. As the shear rate in the interior of the layer is smaller than that near the surface and wall, there will be a plug region in the middle of the layer. The flow consists of three regions: I and II are yielded regions where the shear stress is larger than the yield stress, while III is the unyielded or plug region. The ranges of I, II and III are $[0, z_0]$, $[z_0 + h_0, 1]$, and $(z_0, z_0 + h_0)$, respectively. Here, $0 < z_0 < z_0 + h_0 < 1$.

2.1. Governing equations

The scaled constitutive equation of a Bingham fluid can be written as follows [24],

$$\boldsymbol{\tau} = \mu\dot{\boldsymbol{\gamma}} \Longleftrightarrow \tau > \frac{B}{R}, \tag{2.1}$$

$$\dot{\gamma} = 0 \Longleftrightarrow \tau \leqslant \frac{B}{R}, \tag{2.2}$$

$$\mu = \frac{1}{R}\left(1 + \frac{B}{\dot{\gamma}}\right), \tag{2.3}$$

where $\boldsymbol{\tau}$ is the stress tensor, μ is the dimensionless effective viscosity, $\dot{\boldsymbol{\gamma}}$ is the strain-rate tensor with the form $\dot{\boldsymbol{\gamma}} = \nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}$, $\mathbf{u} = (u, v, w)$ is the velocity, $\tau = \sqrt{\dot{\tau}_{ij}\dot{\tau}_{ij}/2}$ and $\dot{\gamma} = \sqrt{\dot{\gamma}_{ij}\dot{\gamma}_{ij}/2}$ are the

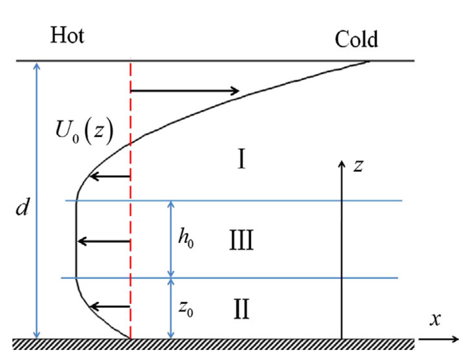

Fig. 1. The schematic of the thermocapillary liquid layer for a Bingham fluid. Here, I and II are yielded regions, III is the plug region, d is the depth of the layer, z_0 is the length of the yielded region II, h_0 is the length of the plug region, U_0 is the velocity field.