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Evolution of natural convection melting inside cavity heated from different sides using enthalpy based lattice Boltzmann method

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ABSTRACT

Melting coupled with natural convection in cavity has always been a hot topic. In this paper, natural convection melting in cavity heated from different sides are numerically simulated using the enthalpy based lattice Boltzmann method. The cavity model of one heated side and three adiabatic sides is employed to achieve totally melting. The dimensionless numbers are: $Pr = 0.02$, $St = 0.01$, $Ra = 25{,}000$ and $Ra = 50{,}000$. Melting when heated from top is conduction melting, melting when heated from right is omitted due to its symmetry with the case when heated from left. The heat transfer and flow characteristics when heated from left and bottom are mainly investigated. Results indicate that, when heated from left and bottom, the melted region always enlarges but the growth of flow velocity is suppressed. In the final stage of melting, the temperature distribution tends to be uniform and the flow inside cavity gradually vanishes. Moreover, the melting efficiency when heated from bottom is significantly decreased by its first stage of conduction melting. As *Ra* increases, that stage is obviously shortened, the melting efficiency is promoted rapidly. And the melting efficiency when heated from bottom exceeds the efficiency when heated from left at around $Ra = 25{,}000$. Besides, melting from different sides are actually the problems of different angles between the heat flux out of the boundary and gravity. The effect of more different angles on the melting efficiency is further discussed. As the angle increases, the average dimensionless velocity also increases. Natural convection inside the cavity becomes stronger, and the melting efficiency gets higher.

1. Introduction

Melting is a physical process that results in the phase transition of a substance from solid to liquid. Nowadays, it widely exists in all kinds of industrial applications, such as metal casting, welding and the prevailing additive manufacturing. It is so important that it always arouses the interests of scientific research. However, analytical solution of this process could not be obtained due to the non-linear term in its governing equation, thus numerical and experimental researches are carried out instead.

Given the complication of melting process, simplified models are often adopted to investigate its fundamental mechanism. Melting coupled with natural convection in cavity, which has two adiabatic sides and another two sides with constant temperature, is the most commonly used model. Usually the hot side is put at left to heat the cavity and initiate the melting. Numerous studies for both numerically and experimentally were conducted, and approximate solutions were summarized [1] when heated from left. The melting front, isothermal lines and streamlines under certain dimensionless numbers have been obtained to act as benchmark solutions. Thus recent studies about natural convection melting in cavity heated from left have been more of verification for their new methods [2,3] and models [4]. And some of them also studied the influences of different shapes of cavity [5], inclined angle [6], magnetic field [7] and so on. However, the flow velocity inside the cavity is barely mentioned among those researches, which would also be a significant supplement for better understanding of its heat transfer and flow characteristics. Researches about melting when heated from bottom are limited, for that heating from bottom could trigger the famous Rayleigh–Bénard flow. Gau [8] and Viskanata [9] conducted a series of melting experiments to investigate the heat transfer characteristics and even visualized the flow of high *Pr* substance melting from bottom, they observed the development of inside flow regime. Gong [10] numerically studied the flow patterns for a wide range of *Ra* for the melting of high *Pr* substance, and it was found that instability of natural convection would be invoked as *Ra* increased. But Mehdi [11] found that for low *Pr* substance, the melting interface was different and it would become asymmetry. Semma [12] found the symmetry

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Nomenclature

c_s	sound speed of the model, $m{\cdot}s^{-1}$
C_p	specific heat capacity, $kJ{\cdot}kg^{-1}{\cdot}K^{-1}$
e	discrete velocity, $m{\cdot}s^{-1}$
F	body force, $kg{\cdot}m{\cdot}s^{-2}$
Fo	Fourier number, $Fo = \alpha t/l^2$
f	density distribution function
f_l	liquid fraction
f^{eq}	equilibrium density distribution function
g	energy distribution function
g^{eq}	equilibrium energy distribution function
$\mathbf{g}$	acceleration of gravity, $m{\cdot}s^{-2}$
h	convective heat transfer coefficient, $W{\cdot}m^{-2}{\cdot}K^{-1}$
H	total enthalpy, $kJ{\cdot}kg^{-1}$
H_l	total enthalpy of the liquid phase, $kJ{\cdot}kg^{-1}$
H_s	total enthalpy of the solid phase, $kJ{\cdot}kg^{-1}$
k	thermal conductivity, $W{\cdot}m^{-1}{\cdot}K^{-1}$
l	characteristic length, m
L	latent heat, $kJ{\cdot}kg^{-1}$
M	number of discrete velocity
Nu	Nusselt number, $Nu = hl/k$
Pr	Prandtl number, $Pr = \upsilon/\alpha$
p	pressure, Pa
Ra	Rayleigh number, $Ra = \mathbf{g}\beta\Delta T l^3/(\upsilon\alpha)$
Ste	Stefan number, $Ste = C_p \cdot \Delta T/L$
t	time, s
T	temperature, K
T_1	heated temperature, K
T_0	melting temperature, K
$\mathbf{u}$	velocity, $m{\cdot}s^{-1}$
$\mathbf{x}$	location of particle

Greek symbols

α	thermal diffusivity of liquid phase, $m^2{\cdot}s^{-1}$
β	volume expansion coefficient, K^{-1}
τ_f	relaxation time of density evolution equation
τ_T	relaxation time of energy evolution equation
υ	kinematic viscosity, $m^2{\cdot}s^{-1}$
ρ	density, $kg{\cdot}m^{-3}$
ω	weight factor
η	melting efficiency

Subscripts

i	direction of discrete velocity

Superscripts

$*$	dimensionless form

Abbreviation

LBM	Lattice-Boltzmann method
TLBM	Enthalpy-based lattice Boltzmann method
HLBM	Total enthalpy-based lattice Boltzmann method

breaking as well, and the reason has not been clarified yet. Recent studies about melting when heated from below are about additives that might enhance the heat transfer [13,14]. But the fundamental principle of melting when heated from bottom still requires further investigation, especially for low *Pr* substance. Besides, all of those researches hardly investigated the whole melting process, due to the limitation of classical melting cavity model. Moreover, melting when heated from different sides actually shares the same governing equation, but only different boundary conditions (more specifically, the difference between them is the angle of heat flux out of the boundary and gravity). They may invoke different physical phenomenon, but in the point of view of engineering, they are both optional approaches for melting. There should be some comparisons between them, but no such research has ever been reported. Here, the melting efficiency, which is defined as the reciprocal value of the dimensionless time that takes to achieve completely melting inside the cavity, is introduced. And the melting efficiency when heated from different sides under different conditions is discussed.

Experimental studies [15,16] concerning about this melting problem require sophisticated devices and strict operating conditions, which are not available in most labs, so that numerical studies are more prevailing these days. Novel numerical methods and models are developed to deal with the problem, such as finite volume method [17,18], finite element method [19], and in particular, lattice Boltzmann method [20]. Jiaung et al. [21] firstly introduced the enthalpy based method (TLBM) into the conduction dominated melting problem. The solid/liquid interface could be tracked automatically by the enthalpy of each node, perfectly solving the moving boundary problem. But in the method, massive iterations were needed to deal with the nonlinear term in the energy equation. And then Chatterjee [22,23] and Chakraborty [24,25] revised the enthalpy model and extended the solution to convection melting and even crystal growth. Huber et al. [26] also modified Jiaung's model and successfully applied it to natural convection melting in cavity. Meanwhile, Huber pointed out that the iteration procedures could be avoided if the relaxation time was properly chosen. Based on TLBM, Huang et al. [27] developed a total enthalpy method (HLBM) which used the total enthalpy to represent the temperature in the energy equation and thus eliminated the nonlinear term. But Luo et al. [28] found that there were nearly no difference on the accuracy of solution between these two methods. In summary, TLBM is a solid and reliable method for melting problems.

Additive manufacturing has received considerable attention in the past few decades. The better understanding of basic metal melting process is the significant footstone to take control of the melt pool during additive manufacturing and promote its technology and efficiency. Based on the reliable enthalpy-based lattice Boltzmann method, the purpose of this study is to investigate the natural convection melting of low *Pr* substance (e.g. metal) inside cavity heated from different sides. The classical cavity model is changed to be one heated side and three adiabatic sides to achieve totally melting. The goals can be listed as: (1) to obtain the basic melting fronts, isothermal lines and especially the velocity distributions when heated from different sides and illustrate the differences; (2) from the point of view of engineering, to compare the melting efficiency under different melting conditions, and further discuss the underlying reason.

2. Physical and numerical models

2.1. Physical and mathematical model

As depicted in Fig. 1, a square cavity filled with solid substance, which keeps at its melting temperature, is to be heated from different sides. And the substance inside is to be melted into liquid phase. The melting temperature is set to be $T_0 = 0$, and the temperature of the heated side is set to be $T_1 = 1$. Boundary conditions are also displayed in the figure. Dimensionless numbers can be listed as $Pr = 0.02$, $Ste = 0.01$, $Ra = 25{,}000$ and $Ra = 50{,}000$. The assump-