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The effects of bottom wall heating on mixed convection of yield stress fluids in cylindrical enclosures with a rotating end wall

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ABSTRACT

Steady-state laminar mixed convection of Bingham fluids in cylindrical enclosures with a rotating top cover has been numerically analysed for the configuration where the bottom cover is kept at a higher temperature than the rotating top cover. The numerical investigations have been carried out based on steady-state axisymmetric incompressible flow simulations for a range of different values Reynolds, Richardson, and Prandtl number given by $500 \leqslant Re \leqslant 3000$, $0 \leqslant Ri \leqslant 1$ and $10 \leqslant Pr \leqslant 500$ respectively. The aspect ratio (i.e. height: radius $= AR = H/R$) of the cylindrical container is considered to be unity (i.e. $AR = H/R = 1$). The mean Nusselt number $\overline{Nu}$ has been found to decrease sharply with increasing Bn owing to flow resistance arising from yield stress, but subsequently $\overline{Nu}$ asymptotically approaches a value of unity, which is indicative of a conduction-driven transport. In addition, the mean Nusselt number $\overline{Nu}$ has been found to increase with increasing Reynolds number due to the strengthening of advective transport. However, the mean Nusselt number $\overline{Nu}$ exhibits a non-monotonic trend (i.e. increases with increasing Ri for small values of Richardson number before showing a weak decreasing trend) with increasing Ri for Newtonian fluid (i.e. $Bn = 0$), whereas $\overline{Nu}$ increases with increasing Ri for small values of Richardson number before becoming a weak function of Ri for Bingham fluids. A step change in the mean Nusselt number has also been observed with an increase in Richardson number for some Bingham number values due to a change in flow pattern. The influences of Prandtl, Reynolds, Richardson, and Bingham numbers on the mean Nusselt number have been explained in detail based on both physical and scaling arguments. The simulation data and scaling relations have been utilised to propose a correlation for the mean Nusselt number, which has been shown to capture the numerical findings satisfactorily for the parameter range considered here.

1. Introduction

The swirling flow produced by rotating one of the end walls of cylindrical enclosures has several engineering applications (e.g. chemical processing, bio-chemical synthesis, polymer processing, food preparation, applications involving magneto-rheological and electro-rheological fluids etc.) for the purpose of augmenting the rate of heat transfer and mixing at relatively small values of Reynolds number. Flows of Newtonian fluids (where the viscous stress is directly proportional to the strain rate) in cylindrical enclosures with a rotating end cover have been extensively analysed in the existing literature [1–9]. The analysis of swirling flows in this configuration for Newtonian fluids was pioneered by the seminal experimental investigations by Vogel [1,2], Ronnenberg [3], Bertela and Gori [4], and these analyses reported vortical fluid motion within the enclosure as a result of the rotation of an end cover. The findings of these analyses [1–4] have subsequently been extended by Escudier [5] who used experimental investigation to demarcate the stability criterion for vortex breakdown in this configuration in terms of aspect ratio (i.e. height to radius ratio H/R) and Reynolds number $\Omega R^2/\nu$ (where Ω is the angular speed and ν is the kinematic viscosity). In addition to these experimental studies, several authors [6–9] numerically investigated this configuration for Newtonian fluids. Lee and Hyun [8] analysed the effects of Prandtl number on heat transfer rate in this configuration and reported significant Prandtl number dependence of the mean Nusselt number in this configuration. Iwatsu [9] analysed the effects of Reynolds and Richardson numbers (in the range of $100 \leqslant Re \leqslant 3000$ and $0 \leqslant Ri \leqslant 1$ for $Pr = 1$) on the flow pattern and heat transfer rate for swirling Newtonian fluid flows in a

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Nomenclature

a	bridging function [-]
a_1, a_2	correlation parameters [-]
AR	aspect ratio, (AR = H/R) [-]
b	bridging function [-]
b_1, b_2	correlation parameters [-]
Bn	Bingham number [-]
c_p	specific heat at constant pressure [J/kg K]
C_T	torque coefficient [-]
e	relative error [-]
Ec	Eckert number [-]
g	gravitational acceleration [m/s^2]
Gr	Grashof number [-]
h	heat transfer coefficient [W/m^2 K]
H	height of cylindrical enclosure [m]
k	thermal conductivity [W/m K]
k_0,k_1	correlation parameters [-]
m_0,m_1	correlation parameters [-]
Nu	Nusselt number [-]
$\overline{Nu}$	mean Nusselt number [-]
Pr	Prandtl number [-]
q	heat flux [W/m^2]
R	radius of cylindrical enclosure [m]
Ra	Rayleigh number [-]
Re	Reynolds number [-]
Ri	Richardson number [-]
T	temperature [K]
U	characteristic velocity scales in radial direction (m/s)
V	characteristic velocity scales in tangential direction (m/s)
V_ϕ	non-dimensional swirl velocity, ($V_\phi = \upsilon H/\alpha$) [-]
α	thermal diffusivity [m^2/s]
β	coefficient of thermal expansion [1/K]
$\dot{\gamma}$	shear rate [1/s]
δ,δ_{th}	hydrodynamic and thermal boundary layer thickness [m]
θ	non-dimensional temperature, ($\theta = (T - T_C)/(T_H - T_C)$) [-]
μ	plastic viscosity [Ns/m^2]
μ_{yield}	yield viscosity [Ns/m^2]
υ	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
τ	shear stress [N/m^2]
τ_y	yield stress [N/m^2]
ϕ	general primitive variable [-]
$\boldsymbol{\Omega}$	angular velocity [1/s]
$\boldsymbol{\Psi}$	stream function [m^2/s]
$\boldsymbol{\Psi}$	non-dimensional stream function, ($\Psi = \psi/\alpha$) [-]

Subscripts

adv	advective
$Bn = 0$	Newtonian fluid case
C	cold wall
$conv$	convective
$diff$	diffusive
eff	effective value
H	hot wall
max	maximum value
r	radial direction
ref	reference value
wf	condition of the fluid in contact with the wall
z	axial direction
ϕ	tangential direction

Special characters

ΔT	difference between hot and cold wall temperature (= $(T_H - T_C)$) [K]
$\Delta_{\mathrm{min,cell}}$	minimum cell distance [m]
r	grid expansion ratio [-]

cylindrical enclosure with a heated rotating top wall for an aspect ratio of unity (i.e. $AR = H/R = 1$). The analysis by Iwatsu [9] revealed that advective transport weakens, whereas diffusive transport strengthens with an increase in Richardson number. All the aforementioned analyses were carried out for Newtonian fluids but heat transfer characteristics of yield stress fluids (i.e. fluids which flow only when a certain stress level is surpassed) in cylindrical enclosures with a rotating end wall received relatively limited attention in the existing literature despite its wide range of applications in chemical and food processing. Most analyses on the non-Newtonian fluid flow in cylindrical enclosures with a rotating cover were carried out for viscoelastic fluids [10,11]. Escudier and Cullen [10] experimentally analysed cylindrical enclosures with a rotating top cover for shear-thinning viscoelastic fluids, and reported that the vortex structure is different from the Newtonian fluid case and an intense toroidal vortex in the vicinity of the rotating cover drives a secondary low intensity vortex. Stokes and Boger [11] proposed a regime diagram for flow stability based on Reynolds and Elasticity numbers for viscoelastic fluids in cylindrical enclosures with a rotating cover. A recent experimental analysis of heat transfer under 'elastic turbulence' of viscoelastic fluids within cylindrical enclosures with a rotating top cover was reported by Traore et al. [12]. It was found by Traore et al. [12] that the heat transfer rate under elastic turbulence might locally increase up to 4 times in comparison to the value in the purely conduction regime due to the secondary irregular motion induced by elastic instabilities in polymer flow with large relaxation times. The influence of shear-thinning on vortex breakdown (observed by Vogel [1,2] and Escudier [5] for Newtonian fluids in the past) in cylindrical enclosures with a rotating cover was experimentally and numerically analysed for non-Newtonian fluids by Böhme et al. [13] where the viscosity was approximated by a power-law in terms of shear rate. Böhme et al. [13] constructed an aspect ratio - Reynolds number (AR - Re) diagram, representing the domain of vortex breakdown for shear-thinning fluids. A few analyses [14–16] have recently concentrated on the rotating disk configuration for yield stress fluids. The mass transfer of yield stress fluids in such applications has recently been investigated by Rashaida et al. [14] and semi-analytical approaches for analysing swirling flow of Bingham fluids over a rotating disk have been proposed by Ahmadpour and Sadegyhy [15] and Guha and Sengupta [16]. However, to date, a recent paper of the present authors [17] is only one paper in the existing literature which deals with both fluid flow and heat transfer characteristics of yield stress fluids in cylindrical enclosures with a rotating heated top wall in spite of its application in many chemical and food industries. Turan et al. [17] have numerically analysed mixed convection of Bingham fluids in cylindrical enclosures with a heated rotating top cover for an aspect ratio (height/radius) of unity (i.e. $AR = 1$) for a range of different values of nominal Prandtl, Richardson and Reynolds numbers given by $10 < Pr < 500$, $0 < Ri < 1$ and $100 < Re < 3000$.

It is possible to have four different configurations for cylindrical enclosures with a rotating end wall which are schematically shown in Fig. 1. A careful analysis of these four configurations