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Chebyshev spectral element method for natural convection in a porous cavity under local thermal non-equilibrium model

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ABSTRACT

An accurate Chebyshev spectral element method is developed to simulate the natural convection in a porous cavity using the local thermal non-equilibrium model. Two horizontal walls of the cavity are adiabatic, while the left vertical wall is heated and the right side wall is cooled. The non-dimensional governing equations are derived based on the Darcy model and Boussinesq assumption. The validation test with exact solution verifies the high accuracy of the algorithm, and the numerical results for natural convection in porous square cavity reach an excellent agreement with reported solutions. Two physical problems are investigated: (1) Natural convection in porous cavity at different aspect ratios with fixed temperature on two vertical walls; (2) Natural convection in porous square cavity with temporal sinusoidal temperature on the hot wall and fixed temperature on the cold wall. The effects of aspect ratio and non-dimensional heat transfer characteristic parameters are analyzed.

1. Introduction

Heat and mass transfer in porous media have long been a topic of practical importance owing to the wide engineering applications, such as biological systems [1], petroleum reservoirs [2–4], geothermal engineering [5], enhanced heat transfer of nanofluid flow [6,7]. So far various numerical methods have been popularly applied for this issue, such as finite volume method (FVM), finite element method (FEM), and lattice Boltzmann method (LBM). Goyeau et al. [8] numerically investigated the double-diffusive natural convection in a porous cavity using FVM, where the computational domain is discretized by a series of control volumes. The governing equations are integrated over the control volumes. Siavashi et al. [9] developed a FVM numerical code to simulate double-diffusive conjugate convection in square cavities filled with porous media including internal thermal and different sources very recently. Nguyen et al. [10] simulated the natural convection in a non-Darcy porous cavity filled with Cu-water nanofluid by FEM. The computational domain is divided into numerous non-overlapped elements, and the Galerkin variational principle is applied to obtain the weak formulation of the solution. Similar method can be found in Ref. [11] studying the steady heat transfer in a porous medium fixed in a vertical annular cylinder, and in Ref. [12] focusing on the natural convection in a porous cavity with linearly heated side walls. LBM based on molecular theory is an attractive and relatively new algorithm. Chen et al. [13] used LBM to simulate the double-diffusive convection in a closed cavity filled with non-uniform porous media. Gao et al. [14] developed a thermal lattice Boltzmann model for natural convection in porous cavity under local thermal non-equilibrium conditions. Very recently, Ghasemi and Siavashi [15–17] successfully applied LBM to numerically study the natural convection in a porous cavity with Cu-water nanofluid, and they developed a parallel computational code. Derakhshan et al. [18] investigated the effects of the Joule heating and viscous dissipation on the electroosmotic flow pattern using LBM. Additionally, Kefayati [19–22] did a series of investigations over the natural convection of non-Newtonian nanofluid in a porous cavity by finite difference lattice Boltzmann method (FDLBM). Through all these publications, it can be seen that the numerical method is a significant tool to investigate heat and mass transfer in porous media. Therefore, developing new schemes with higher accuracy is of great interest to the scientific community.

Recently, spectral method (SM) such as spectral collocation method and Fourier-Galerkin method, excites a huge interest in numerical investigation about the heat and mass transfer in porous media, owing to its spectral accuracy distinguishing themselves from FVM, FEM and LBM mentioned above. Chen et al. [23] applied the Chebyshev spectral collation method to simulate the steady natural convection in a square porous cavity, and the effects of Rayleigh number, the inter-phase heat transfer coefficient, as well as the thermal conductivity ratio were analyzed. Shao et al. [24,25]

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Nomenclature

A	aspect ratio
c_p	specific heat of fluid at constant pressure ($\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$)
D	height of the cavity (m)
E_x	the number of elements along x axis
E_y	the number of elements along y axis
g	acceleration due to gravity ($\mathrm{m \cdot s^{-2}}$)
h	heat transfer coefficient between solid and fluid ($\mathrm{W \cdot m^{-2} \cdot K^{-1}}$)
H	dimensionless scaled value of h
k	thermal conductivity ($\mathrm{W \cdot m^{-1} \cdot K^{-1}}$)
K	permeability of the porous medium ($\mathrm{m^2}$)
$\overline{Nu}$	average Nusselt number
N_x	interpolation degree along x axis
N_y	interpolation degree along y axis
P	pressure (Pa)
Ra	Rayleigh number
T	temperature (K)
T_∞	temperature of environment (K)
t	time (s)
U, V	components of dimensionless velocity
u, v	components of velocity ($\mathrm{m \cdot s^{-1}}$)
W	width of the cavity (m)
X, Y	dimensionless Cartesian coordinates
x, y	Cartesian coordinates (m)
Greek symbols	
α	thermal diffusivity ($\mathrm{m^2 \cdot s^{-1}}$)
β	coefficient of thermal expansion ($\mathrm{K^{-1}}$)
γ	modified conductivity ratio
ε	porosity
Γ	thermal diffusivity ratio
τ	dimensionless time
ξ, η	reference coordinates
υ	fluid kinematic viscosity ($\mathrm{m^2 \cdot s^{-1}}$)
θ	dimensionless temperature
ρ	density ($\mathrm{kg \cdot m^{-3}}$)
Ψ	dimensionless stream function
Subscripts	
f	fluid phase
s	solid phase

developed the Fourier-Galerkin method to numerically study the double-diffusive convection in a saturated porous media, and a new benchmark reference solution was produced. However, there are still some challenges for SM to be applied in complex computational domains. This is an undesirable constrain for engineering applications.

Given this, we would like to develop an accurate Chebyshev spectral element method (CSEM) [26] to numerically simulate the natural convection heat transfer in porous media, combining the advantages of SM and FEM, and thus extending the spectral accuracy into irregular computational regions. More specifically, CSEM has its own significant advantages as an algorithm of high accuracy: (1) Compared with FVM, FEM, and LBM, the exponential convergence rate, namely spectral accuracy, leads to a rapidly improved numerical accuracy with an increased interpolation degree. (2) Compared with SM, CSEM can be applied in a complex computational region by partitioning the domain into many subregions. CSEM has been widely used to investigate the incompressible flow [27–31], natural convection [32–34], and acoustic problems [35–37] in homogeneous media. However, there are scarcely any reports about using CSEM to simulate the flow and heat transfer problems in porous media to the best of our knowledge.

There are two widely used approaches to numerically describe the heat transfer within the porous media, named the local thermal equilibrium (LTE) model (adopted in Refs. [10,12,15,23,38–42]) and the local thermal non-equilibrium (LTNE) model (adopted in Refs. [11,14,43–45]). A brief introduction about the development of the LTE and the LTNE model is well summarized in Ref. [43]. It is known that the LTE model assumes the solid matrix and the saturated fluid are in local thermodynamic equilibrium, such that only one energy equation is needed to depict the heat transfer. As for the LTNE model, two energy equations respectively governing the evolution of temperature for fluid and solid phases are necessary to describe the heat transfer between these two phases. The LTE model has been frequently used for the steady problems owing to its simple formulation. However, for the problems of rapid heat transport or problems with significant differences in thermal conductivity and heat capacity between fluid and solid phases [23], the LTNE model has to be applied. Recently, Torabi and Dickson et al. [46–48] used the LTNE model to investigate the entropy generation and heat transfer of nanofluid within porous media that the solid and nanofluid phases can feature internal heat generations. The performance of a two-dimensional, axisymmetric channel with porous inserts attached to the walls was also analyzed under LTNE model by Torabi et al. [49]. Additionally, Wu et al. [50,51] did an investigation over the effect of LTNE on natural convection in a porous cavity with sinusoidally and partially thermally active sidewalls. Gao et al. [52] carried out a numerical study over the solid-liquid phases change with natural convection in porous media using LTNE.

The aim of this research is to develop an accurate Chebyshev spectral element method to simulate the natural convection in a porous cavity with the LTNE model, which is of fundamental interest to flow and heat transfer problems in porous media. Accurate numerical results can be easily obtained by increasing the interpolation order, showing a spectral accuracy. Although the algorithm is mainly performed in a regular cavity in the following discussion, there is no doubt that it can be expanded into complex computational domains owing to the strategy of subregions. It can be seen from the numerical experiments that CSEM is capable of simulating both steady and unsteady natural convection and heat transfer problems in a porous cavity, and the results for natural convection in a porous cavity at different Rayleigh number and different ratios can serve as reference solutions for this problem. We believe CSEM will be extended to wider applications for numerical investigation of flow and heat transfer in porous media.

2. Physical problem and governing equations

2.1. Physical problem

The fluid-saturated porous media is considered in an enclosed cavity with height D and width W. The solid phase is assumed to be homogeneous, isotropic and non-deformable. The fluid is Newtonian and in local thermal non-equilibrium condition with the solid phase. Additionally, the thermal physical properties of the working fluid are taken to be constant except for the density, which is handled according to the Boussinesq approximation. Furthermore, porosity and permeability are assumed to be constant