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Reproducing kernel particle method for coupled conduction–radiation phase-change heat transfer

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ABSTRACT

Interaction between conduction and radiation during the phase-change process within multidimensional participating media was numerically investigated. The solutions were based on the reproducing kernel particle method (RKPM). Test cases were analyzed and compared to results from analytical solutions and other studies to examine the performance of RKPM. Comparisons indicate that the RKPM is stable and accurate. The effect of thermo-optics effect on the phase-change process was carried out with the refractive index increases/decreases linearly with the temperature. The solidification process involving radiation was studied in a 2D enclosure. The effects of different parameters, i.e., extinction coefficients, scattering albedos, conduction-radiation parameters, and latent heat, on the temperature profiles were investigated, and the results show the parameters have significant effects on the solidification process. In addition, the solidification process during convection-radiation cooling was analyzed in a cuboid enclosure with the effect of various phase transition temperature ranges.

1. Introduction

The phase change process within semi-transparent media is an important research field owing to its relevance in various kinds of engineering applications, like films adopted in solar energy, alloy processing, fiber preparation, optical crystal growth, and nuclear engineering. Not only do movement phenomenon of phase transition interfaces and interactions between radiation and conduction occur in the phase-change process of semitransparent media, but free surface, deformation of the border or even flow phenomenon can occur because of thermal stress; thus, the phase transition process is a complex optic-thermo-stress multi-physics coupling problem. The analytical solutions are limited to uncomplicated examples, and numerical methods are more frequently used to investigate the phase transition problems.

Different numerical approaches have been applied to deal with the combined conductive-radiative heat transfer problems involving phase-change processes, such as the finite element method (FEM) [1,2], the finite difference method (FDM) [3], and the finite volume method (FVM) [4–7]. For conductive-radiative heat transfer, obtaining radiation information by numerical methods is the key to find a solution to the coupled heat transfer problem. The discrete ordinates method (DOM) [8,9], the Monte Carlo method [10,11], the modified differential approximation (MDP0 approximation) [12,13], the discontinuous finite element method (DFEM) [14] and the discrete transfer method (DTM) [15] have been applied frequently to obtain the radiation information. The traditional approaches, however, severely depend on the quality of pre-defined meshes for irregular shapes; thus, mesh rezoning should be required when large deformation occurs. To avoid the drawbacks, some so-called meshfree methods have been proposed.

Meshfree approaches have been applied to solve the coupled heat transfer and phase-change processes successfully. The least squares collocation method (LSCM) [16] and the meshfree local Petrov–Galerkin method [17,18] were proposed to solve coupled conductive-radiative heat transfer within a semi-transparent medium and radiative heat transfer within a gradient refractive index medium. The natural element method (NEM) was employed by Zhang et al. [19,20] based on the least squares Galerkin and natural neighbor interpolation to solve the coupled conductive and radiative problem within a participating medium. Kovačević and Kosec et al. [21,22] employed the radial basis function collocation approach (RBFC) to solve the Stefan problem and thermo-fluid phase change problem in 2D geometries. Singh et al. [23] used an element free Galerkin method to solve phase-change processes during cryosurgery in a trapezoid enclosure.

The smoothed particle hydrodynamics (SPH) [24,25] is a meshfree, Lagrangian particle approach. Liu et al. [26] presented the reproducing kernel particle method (RKPM) that is based on the

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Nomenclature

c_p	specific heat capacity, J/kg K
f_l	liquid fraction
G	incident radiation, W/m^2
H	total enthalpy, kJ/kg
I	radiation intensity, W/m^2 sr
k	thermal conductivity, W/m K
L	latent heat, kJ/kg
M	the number of discrete directions
N	conduction-radiation parameter, $k\beta/4\sigma T_{ref}^3$
n	refractive index
$\boldsymbol{n}_w$	unit normal vector of boundary surface
q_R	radiative heat flux, W/m^2
$\boldsymbol{s}$	unit vector in a radiation direction
T	temperature, K
t	time, s
β	extinction coefficient, $\kappa_a + \kappa_s$, m^{-1}
ε	emissivity
μ_m, η_m, ξ_m	direction cosine in direction m
κ_a	absorption coefficient, m^{-1}
κ_s	scattering coefficient, m^{-1}
ρ	density, kg/m^3
σ	Stefan-Boltzmann constant
τ	optical length
φ	shape function
Φ	scattering phase function
ω	scatting albedo κ_s/β
Ω	solid angle, sr

Subscripts

b	black body
l	liquid phase
m	melting
ref	reference value
s	solid phase
w	wall
0	initial
∞	black surrounding

Superscript

m, m'	index for direction

SPH approach. It has the same merits as SPH, and offers a higher accuracy for particle approximation. As the SHP shape function does not satisfy normalization, it cannot easily create accurate solutions when the distribution of discrete particles is non-uniform. The RKPM improves the shape function of the SPH by introducing a continuous reproducing kernel. The kernel is modified by introducing a correction function that improves accuracy on the boundary of the computational domain. The consistency conditions are obtained by the RKPM kernel function throughout the problem domain.

The RKPM is a harmonious combination of the particle approximation and the Lagrangian formulation. It has the advantages of both the meshless particle method and the Lagrangian method. In other meshless approaches, the meshless nodes are simply interpolated points; however, the advantage of the Lagrangian meshfree approach is that discrete particles can carry material properties, such as the volume, momentum, and energy, and are able to move with the influence of inner interactions and external forces. Approximate points and material properties are simultaneously represented by discrete particles, and such advantages make the RKPM more attractive for optic-thermo-stress multi-physics coupling problems. In the RKPM computation, some particles can be placed along material interfaces and boundaries; therefore, determining the free surface, moving boundary and deformation of the border is not a complicated task. The time history of all field variables at arbitrary positions can be obtained and tracked.

The RKPM has been widely applied in various problems in the engineering technology field, like plane strain rolling problems [27], large deformation of nonlinear elastic and inelastic structures [28,29], metal forming processes [30], elastic-plastic deformation problems [31], multiphase flows with free surfaces [32], and transient heat conduction problem [33,34]. The RKPM has more applications in structural mechanics, cases involving large deformation, and fluid mechanics, and the application of RKPM is relatively mature. It is also successfully used to calculate heat conduction problems. Recently, our collaborators [35] have applied the RKPM to 1D pure radiative transfer. Although the RKPM algorithms of stress, heat conduction, and pure radiation were implemented, the current work is far from sufficient in simulating optic-thermo-stress coupling characteristic in the phase-change process under a unified RKPM framework. To the best of the authors' knowledge, up till now, the RKPM has not been used to solve the multidimensional coupled conductive and radiative heat transfer problem in the phase transition process. Thus, the goals of this study are to further extend the application of RKPM to solve coupled radiation and phase-change heat transfer problems within a multi-dimensional participating medium, and to lay the basis for further development of a unified optic-thermo-stress multi-physics coupling algorithm in the phase change process.

The outline of the present work is as follows. The concept of the RKPM is described in Section 2, and the mathematical formulations and the discretization of governing equations under the RKPM framework are presented in Section 3. In Section 4, the effect of thermo-optics effect on the phase-change process is studied with the refractive index increases/decreases linearly with the temperature, and the solidification processes of spherical corium and aluminum oxide particles are investigated. The Laplace equation is solved to verify the performance and accuracy of the RKPM in an irregular enclosure with Dirichlet boundary conditions. In order to expand the application of the RKPM, a solidification process in a 2D participating medium is analyzed, and the effects of different parameters on temperature profiles are investigated. Then we solved the solidification process of a cuboid enclosure under the mixed boundary conditions with the effect of various phase transition temperature ranges. Finally, conclusions are provided in Section 5.

2. Reproducing kernel particle method

The RKPM is first proposed by Liu et al. [26], and offers a higher accuracy for particle approximation, and it improves the shape function of the SPH by introducing a continuous reproducing kernel. The RKPM is a harmonious combination of the particle approximation and the Lagrangian formulation, and discrete particles can carry material properties and move with the influence of inner interactions and external forces. The RKPM has advantages of processing free surfaces, moving boundaries, and deformation of the border.

Building an approximation for $\hat{u}(x, y, z)$ to $u(x, y, z)$ by applying a corrected kernel is the core idea in the RKPM [36]. The corrected kernel approximation in three dimensions problem is given by