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Heat conduction in cylinders: Entropy generation and mathematical inequalities

Xiaowei Tian, Liqiu Wang *

Department of Mechanical Engineering, The University of Hong Kong, Hong Kong
HKU-Zhejiang Institute of Research and Innovation (HKU-ZIRI), Hangzhou, China

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ABSTRACT

We examine the entropy generation regarding its magnitude and the limit as time tends to infinity and apply the second law of thermodynamics to develop mathematical inequalities with heat conduction in adiabatic cylinders. The former shows a bounded entropy generation if the heat conduction is initiated by the initial temperature distribution, but unbounded if the heat conduction involves a heat source with positive volume average over the cylinder. The latter yields various innovative relations that are useful both for studying differential equations and for examining accuracy of analytical, numerical and experimental results. The work not only builds up the relation between the second law of thermodynamics and mathematical inequalities, but also offers some fundamental insights of universe and our future.

1. Introduction

As an important transport process, heat conduction is governed by the first and the second laws of thermodynamics [1–3]. With the classical Fourier's law of heat conduction as the constitutive relation of heat flux density, the relation between the heat flux density vector and the temperature gradient [1,4], the first law of thermodynamics yields the classical heat-conduction equation whose solution provides the temperature field [1,3]. The application of the Fourier's law of heat conduction will then lead to heat transfer rate and the way to control it [1,3].

Applied to heat conduction, the second law of thermodynamics states that: the entropy generation S_{gen} of a system during heat conduction always increases, or, in the limiting case of a reversible process, remains constant, i.e., $dS_{gen}/dt \geqslant 0$ with t being the time [5,6]. This requires that $S_{gen}(t_2) \geqslant S_{gen}(t_1)$ for all $t_2 \geqslant t_1$. With knowing temperature field from the heat-conduction equation, the entropy generation S_{gen} becomes available [5,6]. Applying $dS_{gen}/dt \geqslant 0$ and $S_{gen}(t_2) \geqslant S_{gen}(t_1)$ can then yield mathematical inequalities and thus solution features of heat-conduction equations [2].

The classical macroscopic definition of entropy provides little on its physical meaning [5–19]. Indeed, our understanding and appreciation of entropy come mainly from its applications in various processes and systems. To address this unsatisfactory issue, the first and second laws of thermodynamics were employed in [19] to prove rigorously the process-independence of the heat exchanged between the environment and a system undergoing a totally reversible process, Q_{0R}^{12}, between State 1 and State 2. Although heat is normally process-dependent, Q_{0R}^{12} is the same for all totally reversible processes between two specified states 1 and 2. We may thus define entropy at arbitrary state 1, as $S_1 = CQ_{0R}^{10}$, where state 0 is the reference state whose energy and entropy are assigned to be zero, and C can be any positive constants [19]. The introduction of C is to recover the classical definition of entropy with C being $1/T_0$ (T_0 stands for the environment temperature). It is however more convenient and desirable for showing the nature of entropy and performing entropy analysis to choose $C = 1$ (no unit such that S with an energy unit). Therefore, the entropy S at any state is actually the part of the system energy that cannot be converted into work even with totally reversible processes.

Energy is conserved by the first law of thermodynamics [1,5,6]. The very essence of entropy is the part of system energy that cannot be transformed into useful work [19]. Any entropy generation will then degrade the quality of energy. It becomes thus significant and relevant to examine dS_{gen}/dt regarding the way to reduce its magnitude and $\lim_{t\to\infty} S_{gen}$ regarding whether it is bounded or not.

The present work aims to develop above-mentioned mathematical inequalities and examine dS_{gen}/dt and $\lim_{t\to\infty} S_{gen}$ with heat conduction in three-dimensional cylinders. Note that such an

* Corresponding author at: Department of Mechanical Engineering, The University of Hong Kong, Hong Kong.
E-mail address: lqwang@hku.hk (L. Wang).

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Nomenclature

a	cylinder radius
a_0^2	thermal diffusivity
C_V	specific heat at constant volume
f	internal source
h	cylinder height
S	total system entropy
S_{gen}	entropy generation
t	time
T	temperature

Greek symbols

ρ	density
δ	δ function
φ	initial temperature distribution
Ω	heat-conduction domain

Subscripts

gen	generation
0	environment
φ	heat conduction driven by initial temperature distribution
f	heat conduction driven by internal source
φf	heat conduction driven by initial temperature distribution and internal source

analysis is very limited in the literature and differs fundamentally from other studies of the second law analysis [20–45]. The latter has been made with various approaches [21,22,24,26,42–45] that include the classical entropy [21,22,24], exergy [26,44] and energy-order [45] analysis. The aim of such conventional analyses is mainly for performance evaluation, weak-component identification or performance optimization by varying geometrical, thermal-physical or dynamic parameters in practical systems/processes [20–45]. The present work promotes innovative applications of heat-transfer studies in distinguishing technologies that are with either bounded or unbounded entropy generation, offering fundamental insights of universe and future, and developing mathematical inequalities. In Section 2, we make analytical derivation of temperature field, entropy generation and its limit, and mathematical inequalities. Our derivation is made for the heat conduction driven by the initial temperature distribution, by the internal source and by the both, respectively, with the more details being given for the first case. In Section 3, we summarize the inequalities developed in Section 2 and the physical implication of the entropy generation and its limit obtained in Section 2. We draw some concluding remarks in Section 4.

2. Temperature field, entropy generation and mathematical inequalities

Consider heat conduction in a cylinder Ω of radius a and height h with constant material properties and specified temperature gradient at the cylinder boundary, the second kind or Neumann boundary condition [1]. As the contribution of nonhomogeneous boundary condition to the temperature field can be represented by source and initial terms [2], we can focus our attention to the following initial-boundary value problem with homogeneous boundary conditions in cylindrical coordinates, shown in Fig. 1, without loss of the generality:

$$\begin{cases} T_t = a_0^2 \Delta T + f(r,\theta,z,t), \Omega \times (0,+\infty) \\ T_r|_{r=a} = 0, T_z|_{z=0,h} = 0 \\ T|_{t=0} = \varphi(r,\theta,z). \end{cases} \quad (1)$$

$$\Omega : 0 < r < a, \quad 0 < \theta < 2\pi, \quad 0 < z < h$$

where t and T are time and temperature, respectively. a_0^2 is the thermal diffusivity. $\varphi(r,\theta,z)$ is the initial temperature distribution over the cylinder. $f(r,\theta,z,t)$ is the rate of heat generation inside the cylinder per unit volume and per unit specific capacity of the material. The heat generation may be due to nuclear, electrical, chemical, gammy-ray, or other sources that may be a function of time and/or position.

2.1. Heat conduction initiated by the initial temperature distribution

For the heat conduction driven by the initial temperature distribution, $f(r,\theta,z,t) = 0$, and Eq. (1) reduces into

$$\begin{cases} T_t = a_0^2 \Delta T, \quad \Omega \times (0,+\infty) \\ T_r|_{r=a} = 0, T_z|_{z=0,h} = 0 \\ T|_{t=0} = \varphi(r,\theta,z). \end{cases} \quad (2)$$

To obtain the solution of (2), consider $T(r,\theta,z,t) = T(t)U(r,\theta,z)$, Eq. (2) thus yields

$$\frac{T'(t)}{a_0^2 T(t)} = \frac{\Delta U(r,\theta,z)}{U(r,\theta,z)} = -\lambda, \quad \lambda : \text{constants}.$$

Therefore

$$T'(t) + \lambda a_0^2 T(t) = 0 \quad (3)$$

$$\Delta U(r,\theta,z) + \lambda U(r,\theta,z) = 0 \quad (4)$$

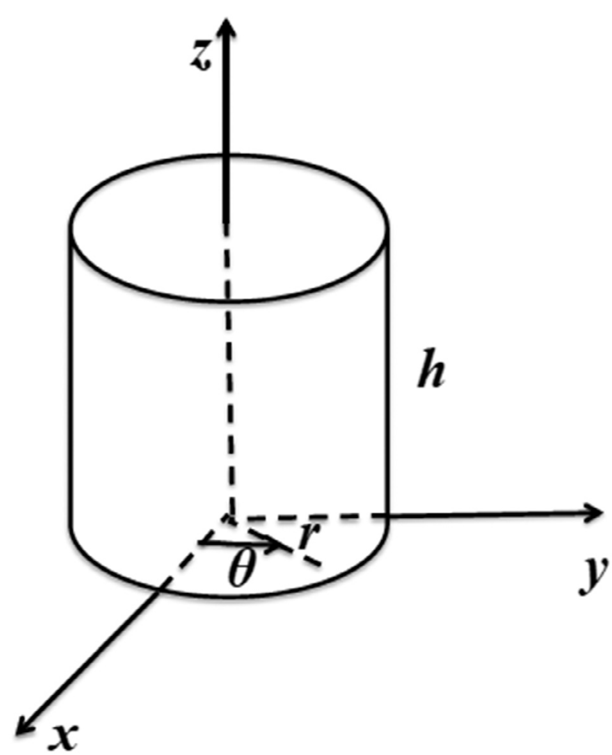

Fig. 1. Heat conduction in adiabatic cylinders and cylindrical coordinate system.