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# Experimental study of the influence of the Prandtl number on the convective heat transfer from a square cylinder

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ABSTRACT

A detailed convective heat transfer study was conducted for a square cylinder subjected to uniform air and water streams at various inclination angles using a wind tunnel and a towing tank approach, respectively. Since the same test object was considered for both fluids, the comparison of the air and water heat transfer data enabled an assessment of the influence of the Prandtl number on the convective heat transfer. The reliability of the test apparatus was checked by means of mean heat transfer results obtained for air for which accurate literature data were available. The new results of the present experimental study demonstrated that the classical assumption $m = 1/3$ for the Prandtl number exponent in heat transfer correlations was only valid in cases where a two-dimensional boundary layer flow governed the convective heat transfer. The classical Prandtl number exponent assumption failed in the case of flow regimes dominated by separation and reattachment. The new experimental data indicated that for such three-dimensional flows virtually higher values should be used for the Prandtl number exponent.

## 1. Introduction

In many heat transfer applications and devices involving single phase flows, the convective heat transfer from objects subjected to forced cross flows is of major importance. These objects are frequently cylinders of circular or non-circular cross sections like square cylinders or inclined plates. A great deal of efforts has been spent on the acquisition of reliable heat transfer correlations for such objects subjected to external flows. For instance, the VDI Heat Transfer Atlas [1] contains a corresponding chapter providing a rather general procedure for obtaining the mean Nusselt Number $Nu_m$ as function of the Reynolds number $Re$ and the Prandtl number $Pr$. The Reynolds number $Re$ is typically defined by means of the inflow or freestream velocity, $u_\infty$, and a suitable characteristic length, $l$. In the case of circular cylinders, the outer radius $R$ (or diameter $d$) might be chosen as natural length scale, and many experimental and theoretical data for mean and local heat transfer phenomena are well available in the literature. For non-circular cylinders, much less information exists. Sparrow et al. [2] reviewed the available mean heat transfer correlations for non-circular and circular cylinders in crossflow. They provided archival correlations for the average heat transfer coefficients. However, a close inspection of the literature indicates a serious lack of knowledge regarding the influence of the Prandtl number $Pr$ on the convective heat transfer. This will be discussed in the following.

The majority of authors propose a dependency of the mean Nusselt number $Nu_m$ on the Prandtl number $Pr$ by means of a simple power law

$$Nu_m = \frac{h_m l}{\lambda} = f(Re) \cdot Pr^{m_m} \tag{1}$$

with an exponent $m_m = 1/3$ up to 0.4 [1,2]. This approach is appropriate in the case of two-dimensional boundary layer flows (see, for instance, [3,4]), but sometimes the laminar boundary layer value $m_m = 1/3$ is also suggested for three-dimensional flows and complex convective heat transfer without experimental proof. This approach might be of some value for covering mean heat transfer phenomena, but it certainly has limits for predicting the *local* convective heat transfer coefficient $h$ at arbitrary locations on blunt objects. This issue is caused by the fact that typically regions of separated flow and hence different convective heat transfer regimes occur in the case of blunt bodies. In Fig. 1, the fluid flow over a square cylinder whose front side is perpendicularly oriented to the external uniform flow with freestream velocity $u_\infty$ is schematically shown. In the case of a normal orientation illustrated by means of Fig. 1 (top), a two-dimensional stagnation flow (SP) occurs at the front side. At the side walls and at the rear side, other flow regimes including separation and reattachment (indicated by the symbol R in Fig. 1) result. In the case of an inclined square cylinder, Fig. 1 (bottom), the both front sides are governed by a two-dimensional

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**Nomenclature**

| | |
|---|---|
| $A$ | area (m$^2$) |
| $B$ | blockage factor (–) |
| $C$ | constant (–) |
| $d$ | square cylinder side length (m) |
| $f$ | function (–) |
| $h$ | heat transfer coefficient (W/(m$^2$ K)) |
| $I$ | electrical current (A) |
| $k$ | exponent (mixed convection) (–) |
| $l$ | characteristic length (m) |
| $m$ | exponent (Prandtl number) (–) |
| $n$ | exponent (Reynolds number) (–) |
| $Nu$ | Nusselt number (–) |
| $\dot{Q}$ | heat flow rate (W) |
| $\dot{q}$ | heat flux (W/m$^2$) |
| $P$ | electrical heat input (W) |
| $Pr$ | Prandtl number (–) |
| $R$ | radius (m) |
| $\mathrm{R}$ | electrical resistance ($\Omega$) |
| $Ra$ | Rayleigh number (–) |
| $Re$ | Reynolds number (–) |
| $T$ | temperature (K) |
| $U$ | voltage (V) |
| $u$ | velocity (m/s) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | inclination angle |
| $\lambda$ | thermal conductivity (W/(m K)) |
| $\nu$ | kinematic viscosity (m$^2$/s) |

*Subscripts*

| | |
|---|---|
| $A$ | air |
| $av$ | side-averaged |
| $C$ | conduction |
| $f$ | film |
| $m$ | mean |
| $MC$ | mixed convection |
| $NC$ | natural convection |
| $rad$ | radiation |
| $w$ | wall |
| $W$ | water |
| $x$ | local |
| $\infty$ | infinity, bulk, or freestream |

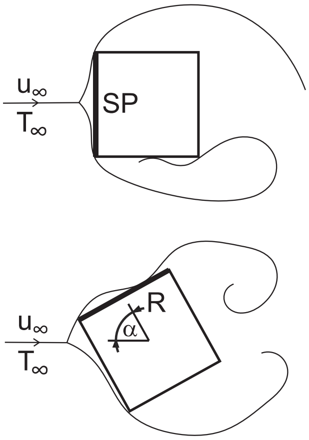

**Fig. 1.** Fluid flow over a square cylinder in a crossflow: normal orientation (top) and inclined square cylinder (bottom). R indicates the location of flow reattachment.

boundary layer flow, and a massive flow separation takes place at the sharp edges. At the rear side of the square cylinder, flow separation and more complex vortex flow regimes exist. Then, the assumption of an identical value for the local and average exponent of the Prandtl number, i.e. $m_x = m_m$, is in principle questionable.

Major experimental efforts has been spent considering only the single fluid air ($Pr = 0.71$). Obviously, such experiments do not permit a statement about the effect of the Prandtl number even in cases where local heat transfer information and data were obtained. As known to the authors, no local heat transfer measurements have been reported for heated square cylinders placed in water crossflow so far. The objective of the present contribution was to assess the influence of the Prandtl number on the mean and the local convective heat transfer from an inclined square cylinder subjected to a crossflow. As illustrated by Fig. 1, the infinite square cylinder including inclination represents a rather canonical configuration for that purpose because here different flow and convective heat transfer regimes simultaneously exist. The effect of the Prandtl number was assessed by performing experiments and heat transfer measurements on the same object at nearly identical Reynolds number levels but for two different fluids. In the present study, low speed crossflows of atmospheric air ($Pr = 0.7$) and pure water ($Pr = 7$) were considered.

## 2. Literature survey

In contrast to the circular cylinder configuration subjected to a crossflow, much less is known regarding non-circular profile shapes like square cylinders (also known as prism or rods). The first systematic mean heat transfer study for non-circular objects conducted Reiher in 1925 [5]. He measured the mean Nusselt number $Nu_m$ against Reynolds number $Re$ for air but his heat transfer data seemed to be rather high. This observation stimulated E. Schmidt to initiate an independent study conducted by Hilpert [6] in 1933. His experiments have been judged as highly precise in the scientific literature for a long time as discussed by Sparrow et al. [2]; his original data and correlation have been quoted by several researchers and have been included in many heat transfer textbooks. Unfortunately, several authors have not presented correctly the results obtained by Hilpert. A serious error regarding the characteristic length $l$ was introduced several decades ago [2]. This issue affected the value of the constant $C_m$ of the mean Nusselt number correlation

$$Nu_m = \frac{h_m\, l}{\lambda} = C_m \cdot Re^{n_m} \quad (\text{for fixed } Pr) \tag{2}$$

but not the exponent $n_m$ of the Reynolds number. This error was identified by Igarashi who performed also very accurate local heat transfer experiments [7–9] for inclined square cylinders subjected to a stream of air. Results from mass transfer measurements have been published by Yoo et al. [10,11]. The results from an experimental study [12] showed a good agreement with the modern literature data [7–11], but all of the modern heat transfer coefficients have been found to be significantly larger (about 40%) than predicted by the classical correlation suggested by Hilpert [6] (but much lower than reported by Reiher [5]). Virtually all heat transfer experiments have been carried out only with air as fluid as pointed out by Sparrow et al. [2]. Therefore, even the influence of the