Modeling heat and mass transfer during ground freezing taking into account the salinity of the saturating fluid

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ABSTRACT

In geotechnical engineering applications, the modeling of artificial ground freezing is primarily aimed at predicting the extent of frozen zone around the cooling sources. This modeling could be more or less complex not only according to the material's texture and the hydro-geological context but also to the salt concentration of the saturating fluid.

Through a thermodynamically consistent framework, a fully coupled heat and mass transfer formulation considering the salinity effect was derived. This formulation was intended to capture the most relevant phenomena of ground freezing encountered in geotechnical applications. Particular attention was given to the phase change problem where appropriate simplifying assumptions were made in order to make the proposed methodology easier to apply in field applications. The proposed approach was validated by means of freezing-thawing laboratory tests, carried out on specimens initially fully saturated with sodium chloride solutions at various concentrations. Good agreement was obtained between the measured and predicted results.

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1. Introduction

The artificial ground freezing technique is used for waterproofing and strengthening soils and soft rocks by decreasing their permeability and increasing their mechanical properties [1]. In the recent years, its use in geotechnical applications has become increasingly popular, especially when the hydro-geological context is so complex that conventional civil and mining engineering methods are either too expensive and not cost-effective or environmentally unacceptable (see, e.g., [2–4]). However, this technique requires the control of the freezing process and the use of numerical tools to predict with accuracy and reliability the evolution of the frozen zones. If the effect of the salt concentration in the saturating fluid is ignored or not correctly modeled, those predictions can be seriously affected because the presence of salt causes the bulk liquid to freeze at lower temperature, depending on the salt concentration [5].

The effect of the salt concentration on the properties of the bulk liquid is not limited to the freezing point depression phenomenon. It also impacts all the thermodynamic properties of both the aqueous solution (density, heat capacity...) and the coexisting phases (latent heat, saturation concentration...). When coupled with the

thermal, fluid flow and mechanical effects, this requires combining knowledge from many engineering disciplines, including chemical and thermo-mechanical engineering.

In chemical engineering, a tremendous effort has been made to provide accurate and complete models for aqueous solutions; see for instance the work of Archer and Carter [6] who developed a wide-range equation of state for the sodium chloride solution on the basis of a model originally proposed by Pitzer et al. [7]. In thermo-mechanical engineering, frozen porous media approaches, both experimental and theoretical, have also been extensively developed. However, there has been little effort in the literature to investigate the effect of the saturating liquid salinity within a unified methodology that can easily integrate the knowledge of the different research communities. Besides, parallel efforts have led to independent developments with considerable inconsistencies in both terminology and methodology. Particularly, we can notice the existence of two research approaches: one modeling explicitly the propagation of the phase-change fronts and other one considering these fronts implicitly through the use of an empirical relationship giving the liquid saturation degree as a function of the porous thermodynamic state. The first approach, despite the fact that in some cases it takes into account the salinity effect (see, e.g., [8–10]), would be very difficult to apply in large scale ground freezing problems since a multitude of fronts can coexist and overlap. As for the second approach, commonly used by civil

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Nomenclature

Main symbols

$\bar{v}_{w\alpha}$	partial specific volume of water in the phase α ($m^3\,kg^{-1}$)
$\bar{g}_{w\alpha}$	partial specific free energy of water in the phase α ($J\,kg^{-1}$)
$\bar{h}_{w\alpha}$	partial specific enthalpy of water in the phase α ($J\,kg^{-1}$)
$\bar{s}_{w\alpha}$	partial specific entropy of water in the phase α ($J\,kg^{-1}\,K^{-1}$)
η_λ	dynamic viscosity of the phase λ (Pa s)
$\dot{\pi}^\gamma$	mass rate of ice formation ($kg\,m^{-3}\,s^{-1}$)
Λ_α	thermal conductivity of the phase α ($W\,m^{-1}\,K^{-1}$)
$\mu_{w\lambda}$	chemical potential of water in the phase λ ($= \bar{g}_{w\lambda}$) ($J\,kg^{-1}$)
v_α	specific volume of the phase α ($m^3\,kg^{-1}$)
ρC	volumetric heat capacity ($J\,m^{-3}\,K^{-1}$)
ρ^α	apparent density of the phase α ($kg\,m^{-3}$)
ρ_α	density of the phase α ($kg\,m^{-3}$)
$\underline{D}_\lambda$	diffusivity tensor associated to the diffusion of salt in the phase λ ($m^2\,s^{-1}$)
$\underline{K}_\lambda$	permeability tensor associated to the filtration of the phase λ (m^2)
$\underline{\underline{\Lambda}}$	thermal conductivity tensor ($W\,m^{-1}\,K^{-1}$)
$\vec{v}_\alpha$	velocity of the phase α ($m\,s^{-1}$)
$\underline{\psi}$	conductive heat flux vector ($W\,m^{-2}$)
$\underline{J}$	diffusion velocity of salt ($m\,s^{-1}$)
V	filtration velocity of the phase λ ($m\,s^{-1}$)
c	salt concentration in the phase λ ($kg\,kg^{-1}$)
C_α	heat capacity at constant pressure of the phase α ($J\,kg^{-1}\,K^{-1}$)
c_{sat}	salt concentration at saturation ($kg\,kg^{-1}$)

D_λ	diffusivity coefficient of the salt in the phase λ ($m^2\,s^{-1}$)
g_α	specific free energy of the phase α ($J\,kg^{-1}$)
h_α	specific enthalpy of the phase α ($J\,kg^{-1}$)
$L_{\lambda\gamma}$	latent heat of phase change on the coexistence curve ($J\,kg^{-1}$)
n	porosity (–)
n_α	volume fraction of the phase α (–)
p_α	pressure of the phase α (Pa)
$p_{\lambda\gamma}$	coexistence pressure at the thermodynamic equilibrium between λ and γ (Pa)
p_c	capillary pressure (Pa)
s_α	specific entropy of the phase α ($J\,kg^{-1}\,K^{-1}$)
S_λ	liquid saturation degree (–)
T	temperature (K)
$T_{\lambda\gamma}$	coexistence temperature at the thermodynamic equilibrium between λ and γ (K)

Subscripts or superscripts α

γ	ice
λ	liquid
σ	solid skeleton

Subscripts k

s	salt
w	water

Special symbols

$\Delta\bar{\varphi}_w$	difference between water partial specific quantities ($\bar{\varphi}_{w\gamma} - \bar{\varphi}_{w\lambda}$)

engineers, soil scientists and hydrogeologists (for a detailed review of this approach, see, e.g., [11–13]), it is often applied to the case of pure water-saturated porous media without clearly specifying the methodology that might be used to explore the general case where the saturating fluid is not a pure liquid. Some models used by hydrogeologists to simulate climate change-induced permafrost thaw already include the salinity effect (see, e.g., [14,15]), however they are based on some simplifying assumptions which are incompatible with the degree of accuracy needed in geotechnical applications, such as constant thermodynamic properties and absence of capillary effects. Some other models developed recently have also attempted to include the salinity effect (see, e.g., [16]), but there is still a need for additional thermodynamic consistency, especially regarding phase change, and for further experimental validation.

The main objective of this paper is to present a modeling approach that deals adequately with the salinity effect and includes the major relevant issues within a thermodynamically consistent framework. The thermodynamics of phase change is directly derived from the liquid and ice Gibbs energies, and the simplifications required to produce the final model and what they involve are rigorously justified. The paper is divided into two parts: the modeling formalism (Section 2) which is based on a macroscopic continuum approach (in the sense of Coussy [17]) and the model validation (Section 3) through comparison with experimental data from laboratory tests, carried out on specimens initially fully saturated with sodium chloride solutions at various concentrations. Note that only the thermo-hydraulic problem coupled with the salinity effects is presented in the first part, and that the laboratory tests aim to validate mainly the salinity effects; the hydraulic aspects were already validated in a previous paper

[18] and the coupling with the mechanical issues will be addressed in a future work.

Notation and nomenclature: Throughout the paper, vectors are denoted by an arrow over the symbol, e.g., $\vec{a}$. Second order tensors are denoted by two lines under the symbol, e.g., $\underline{a}$. The divergence operator is denoted by $\vec{\nabla}\cdot$ and $\vec{\nabla}a$ is the gradient of a. The material time derivative is denoted by a dot over the symbol, e.g., $\dot{a}$. All introduced field variables are assumed to be functions of the position vector $\vec{x}$ and time t. Furthermore, since we are studying a multiphase multicomponent problem, we will make use of subscripts and superscripts to designate quantities of interest. The Greek letters ($\alpha, \beta \ldots$) are used to name the phases, with α for a generic phase. The Latin lower case letters ($j, k \ldots$) are used to name the components, with k for a generic component. The pair of letters kα is used to designate the component k in the phase α. A summary of the main symbols is given in the Nomenclature.

2. Model development

The theory of porous media is an efficient tool for investigating ground freezing processes. With the assumption of local thermal equilibrium (single temperature T) and the concept of volume fractions, adopted in the literature by many authors such as Coussy [17], Nishimura et al. [19] and Vitel et al. [18], the resulting framework is obviously quite simple but is not irrelevant in practice and remains consistent with the level of approximation involved in large-scale geotechnical applications. The proposed model is based on such a framework. Thus, the ground subjected to freezing is modeled as a porous medium consisting of a solid skeleton σ