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Heat conduction constructal optimization for nonuniform heat generating area based on triangular element

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ABSTRACT

A nonuniform heat generating model in a triangular area is built in this paper. Constructal optimizations of the models with constant and discrete variable cross-sectional high conductivity channels (HCCs) are performed by choosing minimum maximum temperature difference (MTD) as optimization objective. Optimal constructs of the triangular element and triangular first order assembly (TFOA) are obtained, respectively. The results indicate that for the TFOA with constant cross-sectional HCCs, the minimum MTD is reduced by increasing the nonuniform heat generating (NUHG) coefficient p, namely the heat conduction performance (HCP) is better when more heat generation is generated near the heat sink. The HCP of TFOA is also improved by increasing its complexity of internal structure. Furthermore, the minimum MTD of TFOA with discrete variable cross-sectional HCCs is reduced by 12.57% than that with constant cross-section. Therefore, the HCP of TFOA can be further improved by adopting discrete variable cross-sectional structures for the HCCs. The optimization results obtained from numerical calculations can provide some theoretical guidelines for the optimal heat dissipation designs of electronic devices.

1. Introduction

With the fast development of science and technology, heat dissipations of various thermal systems become more and more prominent. Especially the designs of electronic component systems are dedicated in the pursuit of low power consumption, high stability, ultra-miniaturization and high integration. Large quantities of heat generations in the electronic components must be transferred timely and efficiently, if not, the performances of which will be seriously worsen. Then, high conductivity channels (HCCs) can be placed in electronic components to enhance heat dissipations, which is viewed as a reliable approach to solve this tough problem. Therefore, a suitable HCC placement to minimize the temperature of electronic device is an important issue in the optimization of heat conduction performance (HCP).

Constructal theory [1–14] is an emerging theory of heat transfer optimization in 1990s, which provides new "geometric philosophies" for the performance optimizations of various transfer processes and systems. Recently, the allied fields of heat transfer optimization and enhancement are making a formidable progress according to Refs. [15–22]. Bejan [23] firstly conducted constructal optimization of a rectangular heat generating volume choosing the maximum temperature difference (MTD) as optimization objective, and obtained the optimal distribution of the HCCs. Ghodoossi and Egrican [24] analyzed the "volume-point" heat conduction problem in an exact way on the basis of abandoning the assumption of heat flux linear distribution in the HCCs according to Ref. [23], and derived an exact solution of optimal constructs for rectangular area. Wu et al. [25] studied the exact solution in-depth. Wu et al. [26] re-optimized the distribution of HCCs, and further decreased the thermal resistance of rectangular area by releasing the constraint that "new order construct is composed of last order optimal constructs". Wei et al. [27] carried out an optimization for rectangular area with discrete cross-sectional HCCs by utilizing MTD minimization, and realized the further reduction of thermal resistance. Ghodoossi and Egrican [28] analytically investigated the temperature distribution in triangular area by adopting minimum MTD as optimization objective, and obtained the corresponding optimal constructs of triangular assemblies. Chen et al. [29] and Xiao et al. [30] re-optimized the heat conduction problems in triangular and disc-to-point areas without the constraint that "new order construct is composed of last order optimal constructs". The results indicated that the MTDs of the two constructs were reduced by 30.26% and 49.3%, respectively. Feng et al. [31,32]

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and Chen et al. [33] further performed the heat conduction constructal designs of triangular and disc-to-point areas at micro and nanoscales, and obtained the corresponding optimal constructs, which were different from the case at conventional scale. Moreover, some authors also investigated the heat conduction problems based on finite element method [34–43] and entropy generation objective [44–49], respectively.

The works of constructal optimizations for heat conduction problems mentioned above are completely carried out on the condition that the heat is generated uniformly, whereas the heat is usually generated nonuniformly in practical electronic device. Cetkin and Oliani [50] established a new model of rectangular area with nonuniform heat generating (NUHG), and theoretically analyzed the influence of the HCC distribution on the MTD of the area by optimizing the angle of Y-shaped HCC. Feng et al. [51] proposed a heat conduction model in a rectangular area with constant and discrete variable cross-sectional HCCs by taking NUHG into account. The optimal construct of rectangular area was achieved in line with the optimization criterion of minimum MTD.

In this paper, based on Refs. [28,50,51], a heat conduction model in a triangular area with NUHG will be established. Constructal optimization of the triangular area will be carried out. The MTD reflects the minimum thermal resistance, and it represents the maximum temperature limitation in the heat generating area, which is significant for the industrial design of electronic devices. Therefore, the MTD will be chosen as optimization objective and minimized by varying the shape of triangular area. Furthermore, the minimum MTD of TFOA with discrete variable cross-sectional HCCs will be obtained based on Refs. [28,50,51]. In addition, the results obtained from constructal optimizations under different conditions of heat generations and HCCs' shapes will be compared.

2. Constructal optimization of triangular element

A triangular element (TE) with NUHG is shown in Fig. 1. The volumetric heat generating rate (HGR) in the triangular area is $q''' \cdot f(x,y)$. Meanwhile, the function $f(x,y)$ of HGR varies with the location. The thermal conductivity of heat generating area is k_0. A HCC (thermal conductivity k_p, width D_0) is placed in the middle of the area so as to enhance heat dissipation. The heat flux in the k_0 material is converged into the HCC, and next discharged into the segment M_0 (temperature $T_{\min}$). In addition, the remaining outer boundaries of the area are entirely adiabatic. One hypothesis is that the thermal contact at the interfaces between the HCC and the heat generating area is perfect, and thus no temperature drop at the interfaces. The length and height of TE are H_0 and L_0, respectively, and its area A_0 $(= H_0 \times L_0/2)$ remains constant. The area fraction ϕ_0 of high conductivity material is defined as the ratio of

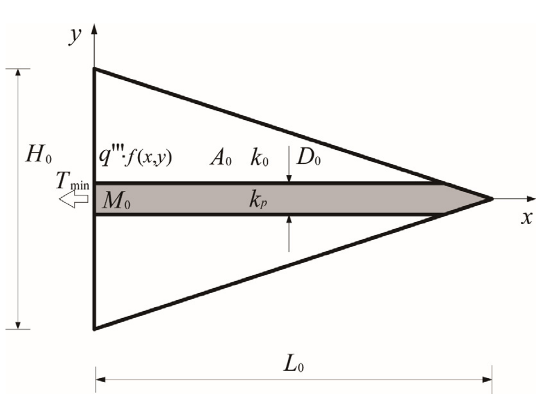

Fig. 1. Triangular element with nonuniform heat generation.

k_p material to the TE, which is fixed as constant too. As for the TE shown in Fig. 1, the area fraction is written as: $\phi_0 = 2D_0/H_0$.

According to Ref. [23], when the constraints of $\phi_0 << 1$ and $H_0/L_0 << 1$ are satisfied, the heat conduction direction in the k_0 material can be assumed to be oriented in the direction of y axis, while that in the k_p material is approximately along the direction of x axis. The partial differential equation of heat conduction in the k_0 material is

$$\frac{\partial^2 T}{\partial y^2} + \frac{q''' \cdot f(x,y)}{k_0} = 0 \tag{1}$$

The boundary conditions for the heat generating area are

$$\frac{\partial T}{\partial y} = 0, \quad y = y_b \tag{2}$$

$$T = T(x,0), \quad y = 0 \tag{3}$$

where $T(x,0)$ denotes the temperature distribution along the HCC, and y_b denotes the vertical ordinate of the points located on the upper and lower borders of the TE, namely

$$y_b = \pm H_0(L_0 - x)/(2L_0) \tag{4}$$

Based on energy balance principle, the partial differential equation in the HCC is

$$k_p D_0 \frac{d^2 T}{dx^2} + 2\int_0^{H_0(L_0-x)/2L_0} q''' \cdot f(x,y)dy = 0 \tag{5}$$

The boundary conditions for the HCC are

$$\frac{dT}{dx} = 0, \quad x = L_0 \tag{6}$$

$$T = T_{\min}, \quad x = 0 \tag{7}$$

In order to obtain the temperature distribution of the TE, it is assumed that the total heat generation is constant and the HGR $q''' \cdot f(x,y)$ is linearly reduced along x axis, i.e., $q''' \cdot f(x,y) = q_0'''(0.05p + 1 - 0.15px/L_0)$, where q_0''' is the heat generating constant and p is the NUHG coefficient [50]. In addition, the HGR is symmetrically distributed on the upper and lower sides of the HCC. Specially, when p is equal to 0, f is equal to 1. In this case, the current model of TE with NUHG in Fig. 1 is simplified to that with uniform heat generation as investigated in Ref. [28].

Integrating Eqs. (1)–(3) leads to the temperature distribution in the k_0 material

$$T(x,y) = \frac{q_0'''}{k_0}\left(0.05p + 1 - \frac{0.15p}{L_0}x\right)\left[-\frac{y^2}{2} + \frac{H_0}{2}y - \frac{H_0}{2L_0}xy\right] + T(x,0) \tag{8}$$

Integrating Eqs. (5)–(7) leads to the temperature distribution along the HCC

$$T(x,0) = \frac{-q_0'''}{k_p D_0}\left[\frac{0.15pH_0}{12L_0^2}x^4 - \frac{(0.2p+1)H_0}{6L_0}x^3 + \frac{(0.05p+1)H_0}{2}x^2 - \frac{H_0 L_0}{2}x\right] + T_{\min} \tag{9}$$

Combining Eqs. (8) and (9) gives the temperature distribution in the TE

$$\begin{aligned} T(x,y) - T_{\min} = {} & \frac{q_0'''}{k_0}\left(0.05p + 1 - \frac{0.15p}{L_0}x\right)\left[-\frac{1}{2}y^2 + \frac{H_0}{2}y - \frac{H_0}{2L_0}xy\right] \\ & + \frac{q_0'''}{k_p D_0}\left[-\frac{0.15pH_0}{12L_0^2}x^4 + \frac{(0.2p+1)H_0}{6L_0}x^3 \right. \\ & \left. - \frac{(0.05p+1)H_0}{2}x^2 + \frac{H_0 L_0}{2}x\right] \end{aligned} \tag{10}$$