International Journal of Heat and Mass Transfer 119 (2018) 343–354

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Optimizing thermal conductivity distribution for heat conduction problems with different optimization objectives

Zi-Xiang Tong, Ming-Jia Li *, Jun-Jie Yan, Wen-Quan Tao

Key Laboratory of Thermo-Fluid Science and Engineering of Ministry of Education, School of Energy and Power Engineering, Xi'an Jiaotong University, No. 28 Xianning West Road, Xi'an, Shaanxi 710049, PR China

ARTICLE INFO

Article history:
Received 31 July 2017
Received in revised form 6 November 2017
Accepted 20 November 2017

Keywords:
Optimization
Heat conduction
Variational method
Thermal conductivity distribution
Average temperature
Quadratic temperature gradient

ABSTRACT

In this paper the optimization of the thermal conductivity distribution for heat conduction enhancement is discussed. Different optimization objectives are considered which include the conductivity weighted quadratic temperature gradient and the weighted average temperature in the whole region or on the heat flux boundary. The adjoint state equations and gradient relations for the optimizations are obtained by the variational method and the 1D and 2D optimization problems are solved to demonstrate the analyses. The analyses show that different objectives are not generally equivalent to each other. When all the temperature boundaries have a same constant temperature, the optimization of the conductivity weighted quadratic temperature gradient has the following equivalences: 1) it is equivalent to the constant temperature gradient relation; 2) it is equivalent to the optimization of heat source averaged temperature in the domain when the heat flux boundaries are adiabatic; 3) it is equivalent to the heat flux averaged temperature on the heat flux boundary when the heat source intensity is zero. Otherwise, the configurations of the optimized temperature and thermal conductivity distributions for different objectives can have large differences. Therefore, the objectives should be carefully chosen when dealing with the heat conduction optimizations.

1. Introduction

The optimization of conductive heat transfer process is important in many applications such as the cooling of the electronic devices. A related issue is to optimize the distribution of a limited amount of high thermal conductivity material on the base material [1], or to optimize the shape of the high conductivity material path for heat conduction [2]. Therefore, the heat transfer can be enhanced and the mass or cost of the material can be reduced. The fast developing additive manufacturing or 3D printing technology also provides opportunities for the optimal design. The complex optimal geometries can be manufactured conveniently by these new technologies [3].

Many methods have been invented for the optimal design of the distribution of high thermal conductivity material. Bejan proposed the constructal theory to design the high thermal conductivity paths for the volume-to-point heat conduction [4,5]. The design started from the smallest building blocks and the blocks were optimally assembled to build larger blocks step by step. The tree-like configurations of the high thermal conductivity material were generated.

The topology optimization is also widely used for the optimization of the heat conduction. In the topology optimizations, different kinds of algorithms and numerical techniques are applied to optimize the material layouts for the objectives and constraints [6]. Here the objective is the starting point of the optimization and a variety of objectives can be used, such as the average temperature or quadratic mean temperature gradient in the domain with heat source [7,8], the heat transfer rate between temperature boundaries per material mass [2], and the volume of the material with the constraint of the heat flux or temperature on the boundary [9].

In the recent decade, Guo et al. [10,11] introduced a new quantity entransy to describe the heat transfer ability. The entransy dissipation can be used to measure the irreversibility of the heat transfer processes. Based on these concepts, the extremum principle of entransy dissipation and the minimum thermal resistance principle were further proposed for the optimization of the heat transfer processes, such as the volume-to-point heat conduction [12], the convective heat transfer in tube or square cavity [13,14], the radiative heat transfer [15] and heat exchangers [16]. However, the concepts of entransy and entransy dissipation still

* Corresponding author.
E-mail address: mjli1990@xjtu.edu.cn (M.-J. Li).

https://doi.org/10.1016/j.ijheatmasstransfer.2017.11.108
0017-9310/

Nomenclature

dS	boundary element (2D: m)
dV	volume element (2D: m^2)
J	optimization objective (1D: $W\,K\,m^{-2}$, 2D: $W\,K\,m^{-1}$)
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
$\bar{k}$	average thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L	augmented functional (1D: $W\,K\,m^{-2}$, 2D: $W\,K\,m^{-1}$)
$\mathbf{n}$	outward-pointing unit normal vector
q	heat flux ($W\,m^{-2}$)
Q	heat source intensity ($W\,m^{-3}$)
T	temperature (K)
w	test function (K)
W	mathematical energy (1D: $W\,K\,m^{-2}$, 2D: $W\,K\,m^{-1}$)
x	space coordinate (m)

Greek symbols

α, β	weight function
γ	total amount of thermal conductivity (1D: $W\,K^{-1}$, 2D: $W\,m\,K^{-1}$)
δ	variation
Γ	boundary
θ	gradient related variable for optimization procedures
$\bar{\theta}$	average value of θ
λ	Lagrangian multiplier ($K^2\,m^{-2}$)
Ω	domain

Subscripts

0	constant boundary condition
b	variable boundary condition
αT	weighted average temperature in domain
QT	heat source weighted average temperature in domain
βT	weighted average temperature on flux boundary
qT	heat flux weighted average temperature on flux boundary
dT	conductivity weighted quadratic temperature gradient
dTq	combination of conductivity weighted quadratic temperature gradient and average temperature at flux boundary
$dTq2$	combination of half conductivity weighted quadratic temperature gradient and average temperature at flux boundary
T	temperature boundary
q	flux boundary
in	inlet heat flux
1	referenced temperature

need further clarification, and the relation between the extremum principle of entransy dissipation and other optimization objectives should be studied.

Since objective is one of the most important points of the optimization, different objectives are compared in this paper, which include the conductivity weighted quadratic temperature gradient, the average temperature in the whole region or on the heat flux boundary. We hope that our study can facilitate further understanding and clarification of the relations and differences between different optimization objectives, and this study can be a reference for further researches in the field of heat transfer enhancement. In the rest of the paper, we first analyze simple 1D optimization problems in Section 2. Then, 2D problems are analyzed in Section 3 and the numerical simulations are used to further demonstrate the analyses. Finally, some conclusions are given in Section 4.

2. One-dimensional problems

2.1. Description of the problems

We start with a 1D optimization example to demonstrate the problem. In order for convenience, the problem is heat conduction per unit of cross-section and the dimension of the problem is 1 m, so coordinate x ranges from 0 to 1 m. The unit of each variable is given in the Nomenclature and in the flowing equations the units are omitted. The heat conduction equation and the boundary conditions for temperature $T(x)$ are given by:

$$\begin{cases} (a): \frac{d}{dx}\left(k\frac{dT}{dx}\right) + Q = 0, \quad 0 < x < 1 \\ (b): T = T_0, \quad x = 1 \\ (c): k\frac{dT}{dx} = q_0, \quad x = 0 \end{cases} \tag{1}$$

Here Q is the heat source intensity and k is the thermal conductivity. q_0 is the heat flux out of the area from boundary $x = 0$. The temperature on boundary $x = 1$ is specified as $T = T_0$.

Before the optimization steps, some basic results are introduced, which will be used in the rest of the paper. Firstly, if Eq. (1a) is multiplied by a test function w with boundary condition $w(1) = 0$ and integrated from 0 to 1, Eq. (1) can be shown to be equivalent to the following weak form [17,18]:

$$\begin{cases} \text{Find } T(x) \text{ that satisfies} \\ T(1) = T_0 \\ \int_0^1 \frac{dw}{dx} k \frac{dT}{dx} dx = \int_0^1 wQdx - (wq_0)|_{x=0} \\ \forall w, w(1) = 0 \end{cases} \tag{2}$$

Secondly, similarly to the principle of minimum potential energy for elasticity problems [17], the variational principle for heat conduction shows that the solution of the strong form (1) is also equivalent to the minimizer of the "mathematical energy" $W(T)$ [18]:

$$W(T) = \frac{1}{2}\int_0^1 k\left(\frac{dT}{dx}\right)^2 dx - \int_0^1 TQdx + (Tq_0)|_{x=0} \tag{3}$$

combined with the boundary condition (b) in Eq. (1). This can be demonstrated by letting the variation of W equal zero as:

$$\begin{aligned} \delta W(T) &= \int_0^1 k\left(\frac{dT}{dx}\right)\left(\frac{d\delta T}{dx}\right)dx - \int_0^1 Q\delta T dx + (q_0\delta T)|_{x=0} \\ &= \left(k\frac{dT}{dx}\delta T\right)\Big|_0^1 + (q_0\delta T)|_{x=0} - \int_0^1 \left[\frac{d}{dx}\left(k\frac{dT}{dx}\right) + Q\right]\delta T dx \\ &= \left[\left(q_0 - k\frac{dT}{dx}\right)\delta T\right]\Big|_{x=0} - \int_0^1 \left[\frac{d}{dx}\left(k\frac{dT}{dx}\right) + Q\right]\delta T dx = 0 \end{aligned} \tag{4}$$

Therefore, both the differential equation (a) and boundary condition (c) in Eq. (1) can be derived from Eq. (4).

Finally, if the governing equation (a) in Eq. (1) is multiplied by T and integrated over [0, 1], we can derive the following relation

$$\begin{aligned} &\left(-Tk\frac{dT}{dx}\right)\Big|_0^1 + \int_0^1 k\left(\frac{dT}{dx}\right)^2 dx - \int_0^1 TQdx \\ &= \int_0^1 k\left(\frac{dT}{dx}\right)^2 dx - \int_0^1 TQdx + (Tq_0)|_{x=0} - \left(T_0 k\frac{dT}{dx}\right)\Big|_{x=1} = 0 \end{aligned} \tag{5}$$

All these terms on the right hand side of the first equation in Eq. (5) can be interpreted according to the concepts of entransy [10]. The first term is the entransy dissipation rate, the second term is the entransy generated by the heat source and the third and fourth terms are the entransy flux out of the domain. Therefore, Eq. (5)