International Journal of Heat and Mass Transfer 118 (2018) 208–222

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Semi-analytic solution of three-dimensional temperature distribution in multilayered materials based on explicit frequency response functions

Haibo Zhang, Wenzhong Wang *, Shengguang Zhang, Ziqiang Zhao

School of Mechanical Engineering, Beijing Institute of Technology, Beijing 100081, PR China

ARTICLE INFO

Article history:
Received 23 March 2017
Received in revised form 25 October 2017
Accepted 29 October 2017

Keywords:
Multilayered materials
Heat conduction
Frequency response functions
Moving heat flux

ABSTRACT

Multilayer coatings have been widely using in a wide range of applications, including industrial, biological and electrical areas, and the thermal distribution in a multilayered material is of great interest. In present paper, the frequency response functions (FRFs) of temperature field under unit point heat flux are derived through thermal conduction equation. The unknown coefficients in the FRFs are assembled in a linear system of matrix equations according to the heat input and continuity condition of heat flux and temperature at each interface; then the coefficients are solved and expressed recursively. Based on the closed-form solution of FRFs, a fast semi-analytical method (SAM) is developed to solve the three-dimensional steady state heat conduction in arbitrary multilayered materials, and there are no limits on the number or the thickness of layers. The temperature fields under different kinds of heat flux in multilayered coatings are studied. Moving heat flux and convection on the surface are also considered.

1. Introduction

Coating/substrate system and functionally graded materials (FGMs) are increasingly used in a wide range of applications to improve the performance of critical components. For such materials, the material properties vary along the depth direction. Used as coatings or interfacial zones, multilayered materials and FGMs can reduce the magnitude of residual and thermal stresses, mitigate stress concentration and increase fracture toughness [1]. Many experimental and numerical results have shown that a properly controlled material property gradient in the FGM can lead to a significant improvement in the resistance to contact deformation and damage [2–5]. In practical applications, the thermal distribution in multilayered materials is of great interest. For example, in mechanical components, temperature shows significant effect on metallurgical microstructure, thermal shrinkage, thermal cracking, residual stresses, and chemical modifications, which greatly influence the performance and reliability of the components [6]. Besides, in the applications of biological, electrical and building areas, the analysis of thermal distribution in multilayered materials is also essential.

A series of research have been devoted to the simulation and investigation of heat conduction in multilayered materials. Although it is difficult to obtain the closed form analytical solutions, thermal conduction in multilayer materials has been solved analytically using various methods. Ozisik studied heat conduction systemically [7], including one-dimensional composite medium. Separation of variables is used much widely [8–13] while searching for associated eigenvalues is the main challenge. Laplace and Fourier transforms [14–18] are also important methods especially for multi-dimensional and time-dependent heat conduction problem. Based on integral transform described by Carslaw [18], Maillet et al. [19,20] developed the thermal quadrupoles method to solve the heat transfer by the use of the electrical analogy in multiblock structures commonly found in electrical circuits and packaged devices, in which the transformed temperature-flux vector at one boundary of the medium is linked to the corresponding output vector at the other boundary by a transfer matrix. For heat transfer in multilayered materials, the transfer matrix is chain product of matrices. When the model contains a large number of layers and three dimensional temperature field is required, it may be time-consuming due to the huge matrix operation. Similarly, Feuillet et al. [21] applied Fourier transform and chain product of matrices to solve the heat transfer in multiblock structure, while the boundary surface should be discretized and iterative calculation is necessary.

Besides, numerical method is increasingly used, including the method of fundamental solutions [22], finite element method (FEM), finite differences [23] or boundary element method (BEM) [24,25]. For a detail description of studies in this subject one can refer to the Ref. [12].

* Corresponding author.
E-mail address: wangwzhong@bit.edu.cn (W. Wang).

https://doi.org/10.1016/j.ijheatmasstransfer.2017.10.118
0017-9310/

Nomenclature

C	influence coefficient obtained by FRFs
$G^{(j)}$, $s^{(j)}$, β	intermediate variables
g	dimensionless surface heat-transfer coefficient, $g = g^{*}l/\kappa$
h_j	dimensionless thickness of layer j, $h_j = h_j^{*}l$
i	pure imaginary, $\sqrt{-1}$
l	characteristic length, [m]
L	number of layers in multilayered material
$M^{(j)}$, $N^{(j)}$	the unknown coefficients in the FRFs for the jth layer
m_x, n_y	the number of grids of computational domain along x and y direction
Pe	Péclet number, $Pe = V^{*}l/\gamma$
Q, Q_n	dimensionless heat input, Q^{*}/q_0; and the updated heat input
S	surface-heated region
T	dimensionless temperature rise; $T = T^{*}\kappa/(l\cdot q_0)$
V	velocity of moving heat flux, [m/s]
κ_j	conductivity of layer j, W/(m K)
γ_j	thermal diffusivity of layer j, [m^2/s]
ω_x, ω_y, ω	variables in frequency domain corresponding to the x and y, $\omega = \sqrt{\omega_x^2 + \omega_y^2}$

In some cases, especially in mechanical elements, the heat source is mainly from frictional contact and the region is usually much small in comparison with the macro contacting body dimensions, thus the assumption of a half-space with layers may be much applicable; the three dimensional model studied for multiblock structures with finite dimensions [9,10,19–21] may be not suitable for tribological system because in this situation the discrete order at the boundary cannot be large due to the matrix manipulation, while for rough surface contact commonly found in tribological system, much dense mesh grid is needed to depict the heat source and to obtain the detailed temperature distribution. Among the studies on multilayered half-space, Shi [26] solved the temperature rise of two and three dimensional single layered half-space through Fourier transform. Particularly, Shi considered the effect of moving source; however, only single layer was involved in the study. The fundamental solution in frequency domain proposed by Simoes et al. [17] presents the solution of three-dimensional multilayered half-space under point heat source; however, the coefficient equations have to be solved at each point and when layer number increases the computational efficiency will become much low. Recently, Dias [27–29] proposed a novel conceptually simple method to solve this problem through recursive images based on the principle of superposition [18]. Although the convection and thermal contact resistance are considered, the method is limited to one-dimension problem up to now. Liu et al. [3] solved the two-dimensional thermoelastic contact problem of FGM, in which Fourier transform and matrix expression are used to solve the temperature field.

It can be pointed out that the most studies have limits in either dimensions or computational efficiency; and the number of layers is also quite limited. The present study focuses on the development of efficient method for solving the heat conduction in multilayered half-space with arbitrary number of layers. It is of the great convenience for applications to develop explicit expressions of the frequency response functions (FRFs) to avoid the tedious numerical procedures [5]. Through the method of FRFs, the stress and displacement fields in multilayered materials have been obtained by [5,30,31]. In present paper, the closed-form FRFs for steady-state temperature field in multilayered materials are derived. It is in explicit recursive form, instead of chain product of matrices or numerically solving a set of equation; and there is no limit on layer number or thickness. Then the influence coefficients are obtained through inverse Fourier transform of the derived FRFs [32,33]. By combining the influence coefficients with semi-analytical method (SAM), which is based on superposition principle, the final temperature rise field under arbitrary distribution of heat input can be obtained. This is particular useful for the cases that the distribution of frictional heat input is complicated when engineering rough surfaces are considered. In present paper the temperature solutions in FGM with different material designs are obtained and the moving heat source and convection on the surface are also considered. It is proven as a fast and accurate numerical method validated with the results from analytical solutions, literature and FEM simulations.

2. Theoretical derivation

2.1. Temperature rise field in frequency domain

A half-space with L coatings is illustrated in Fig. 1, where the coatings are indicated by $j = 1,\ldots, L$, and the half-space is labeled by $L + 1$. All the interfaces are perfectly bonded without heat loss. Note that the origin of the z-axis in each layer is located on its top surface.

The partial differential equation governing heat conduction is given as follows for each layer j in its simplest form by [18],

$$\nabla^2 T^{(j)} = -\frac{V}{\gamma_j}\frac{\partial T^{(j)}}{\partial x}\quad (j = 1, \cdots, L+1) \tag{1}$$

where V is the velocity of heat flux at the top surface, $T^{(j)}$ is the temperature rise and γ_j is the thermal diffusivity of the jth layer material respectively.

At the top surface ($z_1 = 0$), the heat flux is applied as follows

$$-\kappa_1 \left.\frac{\partial T^{(1)}}{\partial z_1}\right|_{z_1=0} = Q \quad \text{for } (x, y) \in S \tag{2}$$

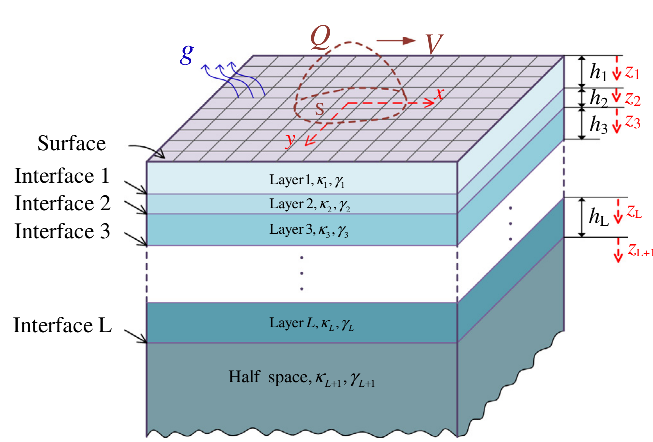

Fig. 1. Schematic of a multilayered material under unit heat input, where the used coordinate system and numbering of layers are defined.