International Journal of Heat and Mass Transfer 118 (2018) 340–348

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A new mechanism for buoyancy driven convection in pulsating viscous flows: A theoretical study

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ARTICLE INFO

Article history:
Received 22 September 2017
Received in revised form 26 October 2017
Accepted 27 October 2017

Keywords:
Pulsating flow
Viscous dissipation
Thermal convection
Linear stability analysis

ABSTRACT

A new mechanism for the onset of thermal convection is proposed. This mechanism is the result of the interaction between a pulsating flow, viscous dissipation, and buoyancy within a channel. The study considers a Newtonian fluid moving inside an infinitely wide horizontal channel bounded by impermeable, rigid plates. The basic flow is characterized by a pulsating pressure gradient. Viscous dissipation acts as an internal heat source which produces a potentially unstable basic temperature gradient. The heat source has a vertical non uniform distribution inside the channel. This configuration is investigated with respect to the onset of buoyancy driven convection. The basic state fields are solved analytically by expanding them in series as functions of the pulsating frequency. In order to perform the linear stability analysis, an arbitrarily small perturbation is superimposed upon the basic state order zero solution. The normal mode method is employed and an ordinary differential eigenvalue problem is obtained. The perturbations were found to have zero angular frequency and thus the resonance phenomena between the basic flow and the perturbations can be neglected. The critical values of the governing parameter are obtained by solving the eigenvalue problem numerically. A growth rate analysis of the possibly unstable configurations relative to the most unstable mode is performed. The present study proves, theoretically, that a pulsating flow can undergo thermal convection. A future experimental study is suggested to validate the proposed instability mechanism.

1. Introduction

We performed a linear stability analysis to investigate the onset of thermal convection in a Newtonian fluid that is driven by a pulsating pressure gradient inside a horizontal channel. This topic may be important in engineering, geophysics and biophysics. Fluctuations of pressure gradient can occur inside different devices and under different geophysical and biophysical circumstances.

Pulsating flows have been studied both theoretically and experimentally. Early studies performed by Womersley [1] and Uchida [2] investigated characteristics of pulsating viscous flows in pipes. Later on, Shemer et al. [3] performed an accurate experimental characterisation of laminar and turbulent pulsating flows in pipes. More recently, Priymak and Miyazaki [4] compared numerical methods for investigating pulsating flows in pipes. They also studied the transition from laminar to turbulent regimes.

Heat transfer associated with pulsating flows was also extensively studied; a short review of the early experimental studies can be found in Havemann and Rao [5]. More experimental data were collected by Habib et al. [6]. Theoretical analysis of convective heat and mass transfer in pulsating flows in pipes and between parallel plates was reported in Siegel and Perlmutter [7], Faghri et al. [8], and Kurzweg [9]. Cho and Hyun [10] reported an accurate numerical solution for the heat transfer characteristics of pulsating flows in pipes; they also implemented a time averaging procedure in order to investigate the behavior of the mean flow. More recently, Guo and Hyung [11] and Moschandreou and Zamir [12] evaluated the Nusselt number in the boundary layer in a circular pipe. Detailed analyses of heat transfer characteristics of pulsating flows for turbulent regimes were reported in Hemida et al. [13] and Wang and Nengli [14]. Nield and Kuznetsov [15] presented analytical solutions that allowed for evaluating the Nusselt number for pulsating flows oscillating with small amplitude in both circular pipes and parallel plate channels.

Stability analysis of pulsating flows was confined to investigations of hydrodynamic instability. An early work by Sarpkaya

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https://doi.org/10.1016/j.ijheatmasstransfer.2017.10.112
0017-9310/

Nomenclature

a	perturbation wavenumber, Eq. (21)
A	pulsation amplitude, Eq. (5)
c	specific heat
$\mathfrak{D}_{i,j}$	stress tensor, Eq. (3)
$\mathbf{e}_z$	unit vector in the z-direction
f	eigenfunction, Eq. (21)
g	gravity
Ge	Gebhart number, Eq. (2)
h	eigenfunction, Eq. (21)
H	channel height
n	Fourier expansion index, Eq. (6)
p	pressure
P	pressure disturbance, Eq. (17)
Pr	Prandtl number, Eq. (2)
t	time
T_l	dimensional reference temperature
T	temperature
$\mathbf{u}$	velocity vector, (u, v, w)
$\mathbf{U}$	velocity disturbance vector, (U, V, W), Eq. (17)
$\mathbf{x}$	position vector, (x, y, z)

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
δz	step-size
ε	dimensionless perturbation parameter, Eq. (17)
η	perturbation growth rate, Eq. (21)
Θ	temperature disturbance, Eq. (17)
Λ	governing parameter, $\sqrt{2\Omega/Pr}$
μ	dynamic viscosity
ν	kinematic viscosity
ξ_i	adjustable parameters, Eq. (23)
Ξ	governing parameter, GeA^2
Φ	viscous dissipation, Eq. (3)
Ψ	streamfunction
ω	perturbation angular frequency, Eq. (21)
Ω	pulsation frequency, Eq. (5)

Superscript, subscripts

$\bar{}$	quantity oscillating as sine, Eq. (6)
$\hat{}$	quantity oscillating as cosine, Eq. (6)
$\breve{}$	quantity divided by A^2
b	basic state
cr	critical value
R	real part
I	imaginary part
$'$	derivative with respect to z

[16] presented a theoretical analysis and some experimental data. Grosch and Salwen [17], Ginsberg [18], Paidoussis and Sundararajan [19], Kerczek [20], Ariaratnam and Namachchivaya [21] reported analyses of hydrodynamic stability utilizing a theoretical approach based on Floquet theory.

The present paper investigates the threshold for the onset of thermal (rather than hydrodynamic) instability. The contribution of viscous dissipation is taken into account. In pulsating flows, viscous dissipation can produce non negligible internal heating, see Gebhart [22]. The pulsating pressure gradient produces velocity gradients close to the channel boundaries due to the no-slip condition imposed at the walls. These velocity gradients, coupled with viscous dissipation, can produce internal heat generation leading to a potentially unstable basic temperature profile. The buoyancy force can then act as a trigger for the onset of thermal convection. The interaction between viscous dissipation and thermal convection in clear fluids and in fluid saturated porous media has been recently studied in Barletta et al. [23,24], Barletta and Nield [25,26], Nield et al. [27].

The main goal of this paper is to investigate how the coupling between viscous dissipation and pulsating flow may affect the onset of thermal convection. The basic state fields are obtained through series expansions with respect to the pulsating frequency of the basic pressure gradient. A linear stability analysis of order zero of the basic state solution is performed. The threshold for the onset of instability is thus studied numerically by employing the shooting method combined with a Runge-Kutta solver for initial value problems. The critical values of the governing dissipation parameter are found. An analysis of the growth rates for configurations with a governing parameter exceeding the critical threshold for the onset of instability is performed. This analysis is carried out for the most unstable mode; the convective cell patterns are reported. Our analysis suggests that the proposed instability mechanism may be relevant to flows of high Prandtl number fluids whose flow is driven by a pressure gradient characterized by a reasonably low, but non-zero pulsation frequency.

2. Mathematical model

A Newtonian fluid is subjected to a pulsating pressure gradient inside a horizontal infinitely wide channel of height H. The channel boundaries are both considered rigid and impermeable. The lower boundary is adiabatic while the upper boundary is kept at a fixed temperature T_l. Our goal is to understand whether or not a pulsating viscous flow may undergo buoyancy driven convection. The choice of the thermal boundary conditions is related to this goal: an adiabatic lower wall and an isothermal upper wall allow us to focus the analysis on the effect of the internal heat generation due to viscous dissipation, see Barletta et al. [28]. A sketch of the channel, which also shows the dimensionless boundary conditions, is given in Fig. 1. The velocity gradients generated by the pulsating flow, coupled with the viscous dissipation, produce an internal heat source that may induce a possibly unstable vertical temperature gradient. The onset of thermal instability of this pulsating flow is studied. The Oberbeck-Boussinesq approximation is applied to

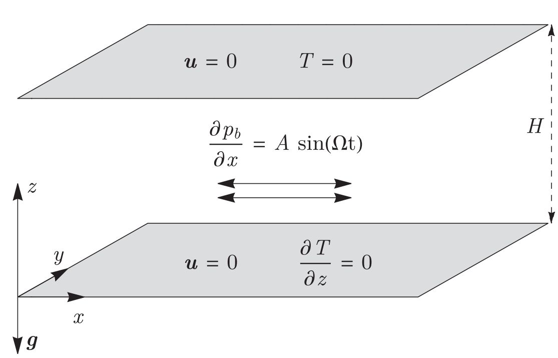

Fig. 1. Sketch of the problem: horizontal infinitely wide channel with impermeable boundaries. The fluid is oscillating horizontally in the x-direction with dimensionless frequency Ω and dimensionless amplitude A. The upper boundary is isothermal and the lower one is adiabatic.