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Characterization of a porous transducer using a capillary bundle model: Permeability and streaming potential prediction

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ABSTRACT

A capillary bundle model is used to evaluate the permeability and streaming potential coupling coefficient of a porous transducer, which is the key element of a fluid-based angular accelerometer. The capillary bundle is specified with structural parameters and a method to transform the particle size distribution into the capillary radius distribution. Together with a theoretical model of zeta potential and specific surface conductance, the model is applied to determine permeability and streaming potential prediction. Three types of transducers are fabricated and experiments validate the proposed model, which is an important part of the sensor model for engineering application.

1. Introduction

Porous media have raised much attention recently in the detection and measurement field owing to its unique fluid and electrokinetic properties. A porous transducer of sintered glass microspheres has been applied in the design of a fluid-based angular accelerometer to transform the liquid pressure in a circular tube into an electrical signal [1–5]. When the porous transducer is immersed in a liquid, the electrical double layer (EDL) exists on the interface between the liquid and the solid [6–12]. A part of the charge in the liquid will be absorbed to the solid surface and the rest stays in the liquid, which is called the diffuse layer. Thus the surface and the liquid become electrically charged. If there is a pressure gradient to force the liquid to flow through the transducer, the movement of the diffuse layer will result in the emergence of the streaming potential. To better understand the relation between the differential pressure and the streaming potential, a simple but workable model is necessary for its characterization.

Diverse mathematical models have been proposed to analyze fluid flow and electrokinetic phenomenon in porous media [5–29]. Numerical methods are always applied to analyze fluid flow at the pore and micro-levels when considering electrokinetics [25–27], but they are time-consuming when applied in engineering, such as digital filter design of the sensor and signal processing. For engineering application, macroscopic properties of porous media are usually sought [5,8,9,14,20–23]. To obtain an analytical model for signal processing of the fluid-based angular accelerometer, a model that is easy to interpret and workable at the macro-level could be a better choice.

The capillary bundle model is one of the most applied models to analyze properties of porous media and has achieved a great success [10–19]. Straight capillary bundles have been used to obtain bulk permeability [14], solute dispersion [17], and the displacement of oil [15]. To deal with more complicated problem, capillary structures with tortuosity [10–13,18] and the interacting capillaries [16,19] have been proposed and have produced satisfactory results. The capillary model also works well in both single-phase [10–14,17,18] and multi-phase flow [15,16,19], and it can couple different physical phenomenon [10–12]. In [10–12], fluid flow and zeta potential were combined together with the capillary bundle model and the multi-phase electrokinetic coupling problem. Because it is convenient to use the capillary bundle model to couple fluid flow and streaming potential without the detailed internal structure and the fluid properties of the porous transducer, this model becomes a workable and useful technique. The experimental pore size distribution is always applied to specify the capillary diameter [13–15], but the measurement of pore size may contaminate or destroy porous medium. In the porous transducer, the particle size of the glass microspheres can be measured before the production of the transducer, which can be used to specify the capillary bundle model instead of the pore size information.

In this paper, we present an equivalent model of the porous transducer comprising a bundle of capillary tubes with a specified tortuosity. By using measured porosity and the particle size distribution (PSD) [4,5], a method to transform PSD into the capillary radius distribution (CRD) is proposed to specify the capillary

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Nomenclature

A	cross-section area of transducer (m^2)
C_{sp}	streaming potential coupling coefficient (V/Pa)
C_a	concentration of acid in liquid (mol/L)
C_f	salinity of solution (mol/L)
d	effective particle size diameter (m)
$\bar{d}_p$	Sauter average diameter of microspheres (m)
e	positive elementary charge (C)
$E(r_c^k)$	kth order original moment of capillary radius (m^k)
E_s	streaming potential (V)
F	formation factor (non-dimensional)
I	ionic strength of bulk pore waters solution (mol/L)
I_s	streaming current (A)
I_c	conduction current (A)
k_b	Boltzmann's constant (J/K)
K	bulk permeability of transducer (m^2)
$K_{(-)}$	dissociation constant for Eq. (9) (non-dimensional)
K_{Me}	binding equilibrium constant for the cation adsorption reaction in Eq. (9) (non-dimensional)
L	length of capillary tube along pressure gradient (m)
L_c	actual path length of capillary tubes (m)
m	cementation exponent (non-dimensional)
M	mass of transducer (kg)
$n(r_c)$	density of capillary tubes with radius r_c (m^{-1})
N_A	Avogadro's number (mol^{-1})
ΔP	differential pressure on transducer (Pa)
Q	flow rate (m^3/s)
r_c	radius of capillary tubes (m)
r_{eff}	effective pore radius (m)
T	temperature (K)
V	total volume of transducer (m^3)
V_f	fluid volume in transducer (m^3)
β_s	electromigration mobility of the adsorbed cations in the Stern layer (m^2/sV)
Γ_s^0	total surface site density on liquid-solid interface (m^{-2})
ε_0, ε_r	vacuum and relative dielectric constant of liquid (F/m, non-dimensional)
ς	zeta potential on liquid-solid interface (V)
Θ	unitless transform value from d to r_{eff} (non-dimensional)
μ	dynamic viscosity of liquid (Pa s)
μ_d, σ_d	mean and variance of PSD (non-dimensional)
μ_c, σ_c	mean and variance of CRD (non-dimensional)
ρ	density of glass microspheres (kg/m^3)
σ_0	bulk electrical conductivity of liquid (S/m)
Σ	total conductance of capillary tubes (S)
Σ_s	specific surface conductance of capillary tubes (S)
Σ_s^{Stern}	surface conductance resulting from the Stern layer of the EDL (S)
Σ_s^{Prot}	surface conductance resulting from proton transfer in the inner part of the EDL (S)
τ	tortuosity in transducer (non-dimensional)
ϕ	bulk porosity of transducer (non-dimensional)

bundle model. Theoretical models for the zeta potential and surface conductance are adopted to explain the electrokinetic property of the porous transducer. The permeability and the streaming potential are predicted and validated by the experiments.

2. Analysis

2.1. Capillary bundle model

The porous transducer is regarded as a bundle of parallel cylindrical capillary tubes with the same tortuosity, $\tau = L_c/L$, where L_c is the actual path length of the tubes, and L is the length of the tube along the pressure gradient (Fig. 1). The radius of the tubes follows a lognormal distribution [30], and the density of the tubes with radius, r_c, is $n(r_c)$. The bulk porosity therefore is,

$$\phi = \frac{V_f}{V} = \frac{\int_{r_{min}}^{r_{max}} \pi r_c^2 n(r_c) \mathrm{d}r_c \cdot L_c}{AL} = \frac{\pi\tau}{A}\int_{r_{min}}^{r_{max}} r_c^2 n(r_c)\mathrm{d}r_c \qquad (1)$$

where V_f and V are the fluid volume and the total volume of the transducer, and A is the cross-section area of the transducer. The flow in the circular capillary can be calculated by Hagen-Poisseuille law, and from Eq. (1), the permeability becomes,

$$K = \frac{Q\mu L}{A\Delta P} = \frac{\int_{r_{min}}^{r_{max}} \frac{\pi r_c^4 \Delta P n(r_c)}{8\mu L_c}\mathrm{d}r_c \cdot \mu L}{\frac{\pi\tau}{\phi}\int_{r_{min}}^{r_{max}} r_c^2 n(r_c)\mathrm{d}r_c \cdot \Delta P} = \frac{\phi}{8\tau^2}\frac{\int_{r_{min}}^{r_{max}} r_c^4 n(r_c)\mathrm{d}r_c}{\int_{r_{min}}^{r_{max}} r_c^2 n(r_c)\mathrm{d}r_c} = \frac{\phi}{8\tau^2}\frac{E(r_c^4)}{E(r_c^2)} \qquad (2)$$

where K is the bulk permeability of the transducer, Q is the flow rate, μ is the dynamic viscosity of the liquid, ΔP is the differential pressure, r_c is the capillary radius, and $E(r_c^k) = \int_{r_{min}}^{r_{max}} r_c^k n(r_c)\mathrm{d}r_c$ denotes the kth order original moment of the capillary radius.

2.2. Streaming potential

The circular tube assumption is convenient not only in the calculation of the permeability, but also in the streaming potential evaluation. The streaming current in a capillary tube is [6,7],

$$I_s(r_c) = -\frac{\varepsilon_0\varepsilon_r \pi r_c^2 \varsigma}{\mu L_c}\Delta P \qquad (3)$$

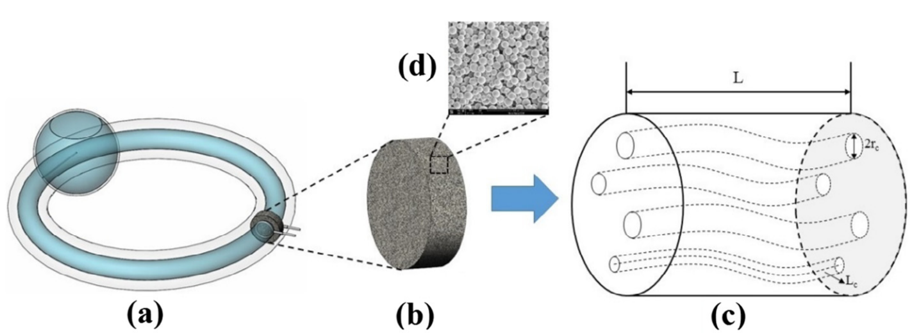

Fig. 1. Porous transducer and capillary bundle model. (a) Liquid-circular angular accelerometer. (b) Porous transducer. (c) Capillary bundle model. (d) Microstructure of the porous transducer.