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Analysis of the characteristic perturbations spectrum of the exact invariant solution of the microconvection equations

V.B. Bekezhanova *

Institute of Computational Modeling SB RAS, Akademgorodok 50/44, Krasnoyarsk 660036, Russia
Siberian Federal University, Svobodny pr. 79, Krasnoyarsk 660041, Russia

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ABSTRACT

The properties of an exact invariant solution of the equations of microconvection of isothermally incompressible liquids have been investigated. The solution describes a stationary fluid flow in a vertical channel. The temperature or heat flux can be given at the solid boundaries of the channel. A classification of the solutions and their physical interpretation are suggested. In accordance with the classification the solutions describe different types of flows. The solution of the stability problem of all classes of flows in the vertical channel with the given temperature on the walls is presented. The structure of the spectrum of small non-stationary spatial perturbations for the model medium (silicon dioxide melt) has been studied, depending on the configuration of the perturbation wave, thickness channel, thermal and gravitational effects. The formation regularities of different types of the thermal and hydrodynamic disturbances have been determined. The interaction of the thermal and hydrodynamic perturbations leads to the formation of various convective structures. Typical patterns of the velocity and temperature perturbations and relations of critical characteristics of the instability are presented, depending on the problem parameters. The most dangerous mechanisms change from hydrodynamic to thermal ones with the variation of the viscous and thermal liquid properties.

1. Introduction

Microconvective phenomena in different systems have been the subject of the detailed investigation in the last few decades (for a review, see [1–5]). The traditional areas of the application of non-isothermal microconvective flows are chemical engineering, materials science, thermophysics. Fluid technologies are applied in the growth of ultra-pure crystals, in microchips for biological systems, in micropumps and micro heat exchangers in power systems and in life-support setups of orbital platforms.

Fluid motion under the joint action of mass, surface forces and thermal loads is the subject of extensive theoretical and experimental investigations. The study of the convective processes is complicated by nonstationarity and nonlinearity. Often, there are some difficulties in the experiments. The first of them is the reconstruction of the conditions in which the investigated phenomenon is observed. Other difficulties are connected with highly accurate measurements of the characteristics and great resource consumption.

Obtaining crystals with high uniformity and structure perfection is a complex technological problem, which stimulates theoretical and experimental investigations. It results in the intensive development of space materials science. The aim of the comprehensive study of the processes responsible for the formation of macro and micro inhomogeneities in crystals is to analyze the influence of the dissimilar physical and chemical factors on the convective processes in melts and solutions, which are used for ultra-pure high-quality crystals [3,6]. It is known that under the terrestrial conditions the gravity forces impede obtaining materials which are uniform in the distribution of components and phases. Thermogravitational convection results in the instability of the crystal growth parameters. In a zero-g field greater advantageous conditions arise for obtaining a homogeneous monocrystal without structural defects. But in space experiments it is specified that in prolonged weightlessness there occur peculiar hydrodynamical processes resulting in arising macro and micro inhomogeneities. Physical reasons for these processes are connected with the action of small forces of gravitational and inertial nature (for a review, see [7,8]).

It is known that the arising cellular perturbations worsen the quality and properties of the obtained pattern [6,9–14]. Hence, the efficiency of the methods of growing ultra-pure crystals from

* Address: Institute of Computational Modeling SB RAS, Akademgorodok 50/44, Krasnoyarsk 660036, Russia.

E-mail address: vbek@icm.krasn.ru

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liquid melts can be enhanced by stabilizing the convective flows of the working fluid. Therefore, special emphasis is placed on the study of the factors causing the formation of spatial defects in crystals. Knowing the critical mechanisms enables one to control the technological process through external effects and facilitates the elaboration of methods of obtaining high quality patterns with the pre-assigned structure and properties both under zero gravity and in the mass forces field.

Investigations in microgravity resulted in the necessity of revising the fundamental constructs of the Oberbeck–Boussinesq model and, as a consequence, in the construction of new exact solutions and study of new problems on the stability of convective flows. As a result of elaborating the Navier–Stokes equations in the Boussinesq approximation, the microconvection equations were obtained by Puknachev [15–17]. The new model allows one to extend the applicability boundaries of the classical models. Within a few years similar equations were suggested by Perera and Sekerka for the investigation of the concentration convection [18]. Mathematical models on the basis of the Navier–Stokes equations were described in [19]. In the framework of the models the convection, heat and mass exchange results were obtained for technical, technological and geophysical applications. Nonstandard treatments for investigating nonlinear hydrodynamics equations, the construction of the exact solution classes of different convection models were described in [20]. In [3,21] the investigation results for gravitational and nongravitational convection under different micro accelerations, including the limit case of theoretical weightlessness were generalized and spatial convective flows realized in the space flight were studied. The mathematical models of convection in weak force fields and on the microscales and analytical investigations of liquid motions performed on the basis of the models were presented in [22,23]. Due to the known analogy between the transfer processes on the microscales and in microgravity [22] the results of the convection study can be highly useful in view of miniaturization of different types of electronic devices.

Dissimilar outer and internal factors affect the character of convective flows in different systems. The intensity of the liquid motion can rise and it can become unstable as a result of the interaction of these factors. As compared with isothermal flows the convective ones have a wide spectrum of disturbances and are characterized by various mechanisms leading to instability. The stable state of equilibrium or motion of the working fluid is the primary condition of the correct functioning of the experimental or industrial equipment using fluidic systems. Therefore, the need for modeling convective flows is dictated by the intention to exactly forecast the behavior of the liquid and to prevent possible crisis phenomena. Thus, in the process of crystal growth various structural defects of the obtained patterns correspond to different types of perturbations. The convective heat and mass exchange causes longitudinal and transverse macro inhomogeneities. The temperature oscillations associated with convective instability in melts induce micro nonuniformities (streaked structure).

Recently, special attention has been paid to the construction and investigation of exact solutions describing the convective flows [20,24]. The exact solutions allow one to investigate in detail the influence of various physical factors on the character of flows, to accurately predict the results of the laboratory experiments and to perform the analysis and find the conditions to ensure the stability.

This is particularly valuable to an exact solution to have a group nature, since precisely a group origination of a solution ensures its physical plausibility and realizability. The microconvection model is based on the exact mass and momentum conservation laws [17,22]. Thus, the basic equations have been formulated on the basis of postulates, which imply the natural symmetry properties of space–time and of a fluid moving in the space. Therefore, the exact solution, that has the group nature, conserves the symmetry properties provided by the derivation of the governing equations. It is not surprising that the microconvection equations possess the group properties and admit a large group of transformations [20]. The main transformation group of the system was calculated in [25] for $\boldsymbol{g} = (0, -g, 0), g = \text{const}$, $\boldsymbol{g}$ is the gravitational vector. On this basis, a number of exact solutions of microconvection equations was constructed [26–29]. An optimal system of the first and second-order subalgebras were calculated by Rodionov in [27,22], respectively. Using the operators of the optimal systems of the first and second-order subalgebras several examples of factor-systems in invariant variables were constructed [27].

Up to the present only one exact solution has been interpreted and studied in stationary [26] and non-stationary [22,29] cases. The solution was applied only for simple geometry, such as a vertical channel. For other geometries exact solutions have not been studied yet. Only numerical modeling of convection was performed in the framework of the microconvection model in the non-stationary case for the following geometries: a long rectangle, a ring domain with solid boundaries, a semicircle with a free boundary and ring domain with a free boundary [29,30]. Comparison of convection regimes and its thermal and hydrodynamic characteristics calculated on the basis of the Oberbeck–Boussinesq and microconvection models was performed in [30,31] by numerical modeling. Both qualitative and quantitative differences for temperature and velocity fields and for trajectories of liquid particles were found.

Some results, concerning the stability of the above mentioned exact solution in the stationary case, have been obtained [32,33]. In [32] the plane perturbations of the exact solution were considered and comparison with results, obtained for analogical problem in the framework of the Oberbeck–Boussinesq model, was performed. Upon that, the Dirichlet boundary conditions were imposed for the temperature function on the rigid walls. In [33] the structure of spatial disturbances of the exact solution was studied in the framework of the Neumann problem. Stability of equilibrium in a plane horizontal channel with solid walls and with a free boundary has been studied in [34,35]. Comparison of the critical characteristics of the linear stability of some flows and equilibrium configurations obtained in the framework of microconvection and the Oberbeck–Boussinesq models is presented in [36]. Decrease of the threshold characteristics of stability was revealed for the microconvection model.

In the present work the exact invariant solution of the microconvection equations describing the convective flow in the vertical channel at a given temperature on the channel walls is investigated. The properties of spatial characteristic perturbations of the basic flow and the influence of the problem parameters on their structure and mechanisms leading to the change in the flow pattern are investigated in the frame of the linear theory. The results would allow one to solve the problems of convection suppression in melts in view of the improvement of the crystal microuniformity and to define the possibilities to control the convective processes by changing the geometry of the volume filled with the melts, way of heat supply and gravitational effect.

2. General equations and governing parameters

For the description of the convective flows the microconvection equations of the isothermally incompressible liquid are used

$$\operatorname{div} \boldsymbol{w} = 0, \tag{2.1}$$

$$\boldsymbol{w}_t + \boldsymbol{w} \cdot \nabla \boldsymbol{w} + \beta\chi(\nabla\theta \cdot \nabla \boldsymbol{w} - \nabla \boldsymbol{w} \cdot \nabla\theta) + \beta^2\chi^2(\Delta\theta\nabla\theta - \nabla|\nabla\theta|^2/2) = (1 + \beta\theta)(-\nabla q + \nu\Delta \boldsymbol{w}) + \boldsymbol{g}, \tag{2.2}$$