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Multiquadric RBF-FD method for the convection-dominated diffusion problems base on Shishkin nodes ☆

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ABSTRACT

In this paper, a new hybrid scheme based multiquadric radial basis function-generated finite difference (RBF-FD) method with Shishkin nodes is proposed to solve stationary convection-dominated diffusion problems, i.e., the boundary layer problems, which combines the midpoint upwind scheme on the coarse grid with a standard central scheme on the fine grid. Numerical results demonstrate the hybrid scheme with Shishkin nodes has an absolute advantage in accurate over standard upwind scheme with equidistant nodes. Moreover, by varying the shape parameter in multiquadric RBF-FD method, the convergence rates of numerical solutions have been further improved compared with the corresponding FD method.

1. Introduction

In this paper, we consider the following steady convection-dominated diffusion equation

$$\begin{aligned} & -\varepsilon\Delta u+\mathbf{b}(\mathbf{x})\cdot\nabla u=f(\mathbf{x}), \quad \mathbf{x}\in\Omega, \\ & u(\mathbf{x})=g(\mathbf{x}), \quad \mathbf{x}\in\partial\Omega, \end{aligned} \tag{1}$$

where $\varepsilon>0$ and $\mathbf{b}(\mathbf{x})=[b_1(\mathbf{x}),\ldots,b_d(\mathbf{x})]$ denote the diffusion and convection coefficients, $u(\mathbf{x})$ is the variable of interest, the function $f(\mathbf{x})$ is source term, and Ω denotes a bounded domain in R^d (where d is dimension). The convection-diffusion equation describes the combination of two dissimilar physical phenomena, convection and diffusion, and plays a very significant role in fluid flow, heat transfer problems and semiconductor device simulation. When the convective process is dominate over diffuse process, this problem is also a boundary layer problems, i.e., the singularly perturbed boundary problems [11,15]. In this case most standard numerical methods of problem (1) get contaminated due to spurious oscillations and numerical diffusion, for example, compact difference scheme, characteristic finite difference (FD) streamline diffusion method, block-centered characteristic FD method, characteristic variational multiscale method, local discontinuous Galerkin finite element method, etc. [16–25].

It is generally known that a simple and efficient way to solve problem (1) is using standard upwind scheme based FD method. More recently, this scheme and some improved strategy are widely used in local RBF method for solving convection (dominated) diffusion equations [3–9]. In fact, the local RBF method can also be considered as a generalization of the classical FD method to scattered node layouts. It is the same in essence for RBF-FD method [1,2,28]. Taking the one-dimensional case for example, the convention strategy is using the central scheme approximation to the diffusive term and the upwind scheme approximation to the convective term [5], i.e.

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$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_i} = w_{i-1}^{(2)} u_{i-1} + w_i^{(2)} u_i + w_{i+1}^{(2)} u_{i+1}$$

and

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_i} = w_{i-1}^{(1)} u_{i-1} + w_i^{(1)} u_i,$$

where $u_i = u(x_i), x_i$ is the nodes in influence domain and $w_i^{(1)}$ or $w_i^{(2)}$ is unknown weight. In [6], the diffusion and convection terms are both discretized over the upwind influence domain, i.e.

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_i} = w_{i-2}^{(2)} u_{i-2} + w_{i-1}^{(2)} u_{i-1} + w_i^{(2)} u_i$$

and

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_i} = w_{i-2}^{(1)} u_{i-2} + w_{i-1}^{(1)} u_{i-1} + w_i^{(1)} u_i.$$

With the diffusive term approximated by central scheme, a different approach was taken by Chandini and Sanyasiraju [4], who applied the combination of central and upwind scheme approximation to convection term such that the later is employed in the regions where solution gradients are high and the former is used in all the other regions. Furthermore, Yun and Hon [7] proposed a weighted discretization scheme to approximate convection term with the follow form

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_i} = \theta[u_x(x_i)]^U + (1-\theta)[u_x(x_i)]^C,$$

where superscripts U and C denote the $u_x(x_i)$ is approximate by upwind scheme and central scheme respectively, $0 \leqslant \theta \leq 1$. While the various ways of discretizing the differential equation each have their advocates based common structured nodes, the choice of node is just as important. Recently, Qiao et al. [26] presented Wendland RBF-FD method for the time fractional convection-diffusion equation. Li et al. [27] proposed an effective h-adaptive RBF-FD method by using the thin plane spline RBF augmented with additional polynomial functions for the high-dimensional convection-diffusion equation.

So in the present work, we consider a new hybrid discrete scheme base on a special piecewise equidistant nodes (Shishkin nodes [11,12]) for Eq. (1). The hybrid scheme approximation to convection term with midpoint upwind scheme on the coarse mesh and standard central scheme on the fine mesh respectively, which has been successfully used in the FD method [13–15]. In [13], the midpoint upwind scheme is analyzed on the Shiskin mesh and precise convergence bounds are obtained, which show that the scheme is superior to the standard upwind scheme. And the hybrid scheme on the same Shishkin mesh is proved to achieve even better convergence behavior. In view of this, we directly applying the hybrid scheme to the multiquadric RBF-FD method. Numerical results indicate that this hybrid scheme based multiquadric RBF-FD gives higher accurate numerical solutions without non-physical oscillation.

This paper is organized as follows. In Section 2, we deduce the multiquadric RBF-FD formulas in detail. In Section 3, we present three numerical discrete schemes base on FD method and obtain the corresponding schemes based multiquadric RBF-FD in the same way. The results and analysis of numerical examples are presented in Section 4, and we summary the paper in the last section.

2. Multiquadric (MQ) RBF-FD formulation

In this section, the MQ RBF-FD formulas are deduced in detail. Suppose $u(\mathbf{x})$ is a function and $\mathscr{L}$ is a differential operator defined in a domain. Let $X_k = \{\mathbf{x}_1, \ldots, \mathbf{x}_{n_k}\}$ is the neighborhood of the center $\mathbf{x}_k$ with n_k centers, and $\mathscr{L}u(\mathbf{x}_k)$ expressed approximately as a linear combination of the values of u at the n_k centers, so that

$$\mathscr{L}u(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_k} \approx \sum_{i=1}^{n_k} w_i u(\mathbf{x}_i), \tag{2}$$

where w_i is the unknown weight. On the other hand, MQ RBF $\phi_j(\mathbf{x}) = \sqrt{1 + c^2 ||\mathbf{x} - \mathbf{x}_j||_2^2}$ and a polynomial $p(x) = 1$ are used to approximate $u(\mathbf{x})$, then we have

$$u(\mathbf{x}) = \sum_{j=1}^{n_k} \lambda_j \phi_j(\mathbf{x}) + \lambda_{n_k+1} 1, \tag{3}$$

with additional requirement

$$\sum_{j=1}^{n_k} \lambda_j 1 = 0, \tag{4}$$

where c is the shape parameter and λ_j is the interpolation coefficient. Accordingly, we can rewrite (2) as

$$\mathscr{L}u(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_k} \approx \sum_{i=1}^{n_k} w_i u(\mathbf{x}_i) + w_{n_k+1} \sum_{j=1}^{n_k} \lambda_j 1. \tag{5}$$

Substituting (3) into (5) yields

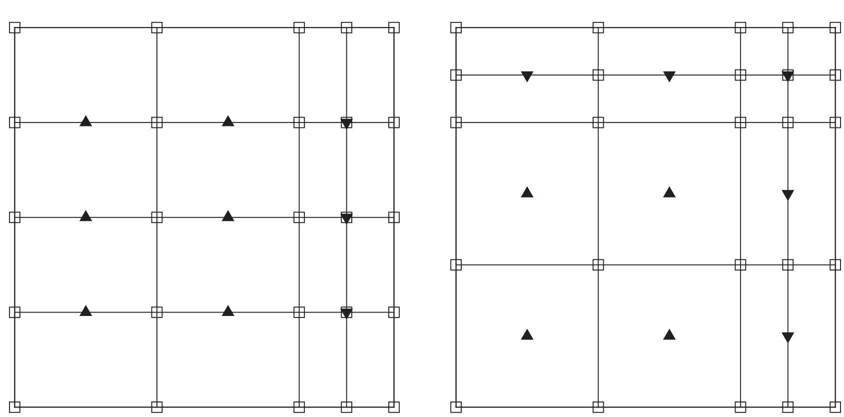

Fig. 1. Shishkin nodes (□), nodes for $b(\mathbf{x})$ and $f(\mathbf{x})$ with midpoint upwind scheme (▲), nodes for $b(\mathbf{x})$ and $f(\mathbf{x})$ with central scheme (▼). Left: single boundary layer; Right: double boundary layer.