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A multiple model adaptive inverse method for nonlinear heat transfer system with temperature-dependent thermophysical properties

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ABSTRACT

A multiple model adaptive inverse (MMAI) method is proposed to estimate the boundary heat flux distribution of nonlinear heat transfer system with temperature-dependent thermophysical properties. In the temperature space of system, the nonlinear heat transfer system is divided into several linear subspaces, and for each subspace, the linear prediction submodel of the temperature at the measurement point is established. Furthermore, according to the instantaneous matching degree between each prediction submodel and the actual heat transfer system, the prediction submodels then are weighted and synthesized to gain the global prediction model of the nonlinear heat transfer system. Finally, based on the global prediction model, the boundary heat fluxes are simultaneously estimated through rolling optimization. Numerical experiments are performed to study the effects of system nonlinearity and measurement errors on the inversion results. Comparisons with the existing dynamic matrix control inverse method and the adaptive sequential function specification method are also conducted, and they all show the validity of the inverse method established in this paper.

1. Introduction

The estimation of internal characteristics or thermal boundary conditions of heat transfer system from the partial observable information is typically called the inverse heat transfer problem (IHTP). The IHTPs are widely used in scientific research and technical fields. In recent years, researchers have done much research on a variety of IHTPs in many fields, such as power engineering, aerospace engineering, metallurgical engineering, bioengineering, etc. [1–7].

The IHTP is a typical ill-posed problem [8]. Over the past few decades, the IHTP and its application research have made great progress and a variety of valuable methods have been developed for the solution of IHTP, for example, the regularization methods [9–13], the conjugate gradient method (CGM) [14–16], the decentralized fuzzy inference (DFI) [3,4,17,18], the stochastic optimization methods [19–21]. Moreover, the sequential function specification method (SFSM) proposed by Beck et al. [8] has been widely used in the unsteady inverse heat conduction problems [6,22,23].

Recently, Woodbury et al. [10,24] studied the structure of Tikhonov regularization method and developed a Tikhonov digital filter solution. This filter form solution has the advantages of simplicity, continuous operation and can be used for near real time heat flux estimation. The technique is further developed by Najafi et al. [25,26] for the solution of inverse heat conduction problems in one-dimensional multi-layer mediums and a two-dimensional plate with multiple heat fluxes at the surface.

In most of the practical engineering problems, thermophysical properties are temperature dependent. As a result, the heat transfer system has obvious nonlinear characteristics, especially if the temperature change in a region is large. The estimation of the unknown boundary conditions for the nonlinear IHTP is more complicated than those for the linear IHTP [27–29]. Therefore, it is of great significance on the study of nonlinear IHTP by looking for effective inverse method with good adaptive ability.

Some results have been accumulated for the nonlinear IHTPs [7,30–35]. Daouas and Radhouani [30] adopted the extended Kalman filter algorithm along with the fixed-interval smoothing technique to solve a transient nonlinear IHTP. García et al. [31] developed a sequential singular value decomposition method for the two-dimensional and non-linear IHTP in irregular-shaped bodies with temperature-dependent thermal properties. By linearizing the nonlinear system, and calculating the sensitivity coefficients at each time step, Beck [32] proposed an adaptive sequential function

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Nomenclature

$\boldsymbol{A}$	prediction matrix
c	heat capacity, J/(kg K)
$\boldsymbol{d}$	the operation matrix for taking the first element of a vector
$\boldsymbol{E}$	temperature deviation vector, °C
I	the number of discrete space grids
J	objective function
L	length of heat conduction body, m
M	linear heat transfer model
N	the number of prediction submodels
q, $\boldsymbol{q}$	heat flux, heat flux vector, W/m^2
$\boldsymbol{Q}$	heat flux matrix, W/m^2
r	number of future time steps
$\boldsymbol{R}$	regularization matrix
t	time, s
T, $\boldsymbol{T}$	temperature, temperature matrix, °C
w, $\boldsymbol{W}$	prediction submodel weight coefficient, weight matrix
X	step response function
x, y	spatial coordinate, m
Y, $\boldsymbol{Y}$	measured temperature, measured temperature matrix, °C

Greek symbols

α	regularization parameter,
$\boldsymbol{\alpha}$	regularization parameter matrix
Δq	heat flux increment, W/m^2
Δt	time step size, s
η	average relative error of the estimated heat flux
θ	characteristic temperature, °C
λ	thermal conductivity, W/(m K)
ρ	density, kg/m^3
σ	standard deviation of measurement error, °C
ϕ	step response coefficients
ω	random number
$\boldsymbol{\Omega}$	prediction submodel set

Superscripts/Subscripts

exa	exact value
k	the current moment
l	index of the heat flux
m	index of the measurement point
n	time index
s	index of the submodel
0	initial value

Decorations

$\sim$	values corresponding to the prediction submodel
–	the predicted values without heat flux increment obtained by prediction submodel
$\wedge$	values corresponding to the adaptive prediction model
$\rightarrow$	the weighted predicted values without heat flux increment

specification method (ASFSM) for the nonlinear IHTPs. Yang and Chen [7] adopted an inverse algorithm based on the CGM and the discrepancy principle to estimate the unknown space and time dependent heat flux of the disc brake system whose thermal conductivity is a function of temperature. Czél et al. [33] proposed an improved genetic algorithm to characterize the temperature-dependent conductivity of a solid material. Khajehpour et al. [34] presented a domain decomposition method for the inverse analysis of nonlinear transient heat conduction problems. The idea of the proposed method is to split the original domain of the problem into a few sub-domains and convert the given inverse problem into several simpler problems. More recently, Cui et al. [35] presented a modified Levenberg–Marquardt algorithm by introducing the complex variable differentiation method for sensitivity analysis, and then the multi-parameters of boundary heat flux are simultaneously recovered by solving the transient nonlinear IHTPs.

For the inverse problem of linear heat conduction system, Wang et al. [36,37] established an inverse method based on the dynamic matrix control (DMC), which can effectively enhance the reliability of inversion results. However, for the nonlinear heat transfer system, as the response function of the system is closely related to the boundary conditions and temperature distribution, it is difficult to establish the prediction model of the measurement point temperature, which makes the DMC method difficult to solve the nonlinear IHTPs.

The multiple model approach decomposes the complex nonlinear system, and uses several simple local linear models to approximate the nonlinear process [38–40]. Using the mature theory of linear system to solve the complex problem of nonlinear system, is an effective way to deal with nonlinear problem. The above-mentioned idea has a good reference significance for studying the nonlinear IHTPs.

For the nonlinear heat transfer system with temperature-dependent thermophysical properties, a multiple model adaptive inverse (MMAI) method by combining the predictive control method with the multiple model approach is proposed in this paper. In the temperature space of system, the nonlinear heat transfer system is divided into several linear subspaces, and for each subspace, the linear prediction submodel of the temperature at the measurement point is established. Furthermore, according to the instantaneous matching degree between each prediction submodel and the actual heat transfer system, the prediction submodels then are weighted and synthesized to gain the global prediction model of the nonlinear heat transfer system. Finally, based on the global prediction model of the nonlinear heat transfer system, the boundary heat fluxes are simultaneously estimated through rolling optimization.

In this paper, the boundary heat fluxes of a two-dimensional nonlinear heat transfer system are estimated by simulated experiments. The effects of system nonlinearity and measurement errors on the inversion results are discussed, and the comparisons with the DMC inverse method and ASFSM are also conducted. Results show that the MMAI method established in this paper can effectively estimate the boundary heat flux of the nonlinear heat transfer system, and has good adaptive ability.

2. Nonlinear heat transfer system and its inverse problem

For the two-dimensional unsteady heat conduction system shown in Fig. 1, the governing equation of temperature $T(x, y, t)$ and the associated initial condition and boundary conditions are stated as follows:

$$\rho c(T)\frac{\partial T(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T(x,y,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T(x,y,t)}{\partial y}\right] \times 0 \leqslant x \leqslant L_x,\ 0 \leqslant y \leqslant L_y,\ t > 0 \tag{1a}$$

$$T(x,y,t) = T_0(x,y) \quad 0 \leqslant x \leqslant L_x,\ 0 \leqslant y \leqslant L_y,\ t = 0 \tag{1b}$$

$$-\lambda(T)\frac{\partial T(x,y,t)}{\partial y} = 0 \quad 0 \leqslant x \leqslant L_x,\ y = 0,\ t > 0 \tag{1c}$$