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Integral equation solutions using radial basis functions for radiative heat transfer in higher-dimensional refractive media

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ABSTRACT

A collocation method based on radial basis functions (RBFs) is applied to solve the integral equations of intensity moments for radiative heat transfer in scattering media with spatially varying refractive index (VRI). Since the method does not require predefined meshes, it can be readily applied to the problem with irregular geometry. Efficient codes of the collocation method with discrete ray tracing are developed. The codes are applied to analyze radiative equilibrium in a semicircular medium with an inner circular boundary. Since rigorous solutions on radiative equilibrium in three-dimensional refractive media are seldom reported, we also apply the present method to analyze radiative heat transfer in cubic media with VRI. The results obtained by using multiquadric RBFs for cases with various optical sizes and boundary conditions are presented. Comparisons of the present results and those obtained by Monte Carlo discrete ray tracing simulation show a good agreement; the discrepancy between the results of the two methods decreases with the increase of the distinct data points used. The present results also show that the temperatures of the cases with a diffusely reflecting semicircular surface are larger than those of the cases with a black semicircular surface and a larger variation of temperature may be observed in the cubic medium with a larger optical thickness.

1. Introduction

Radiative heat transfer in media with a spatially varying refractive index (VRI) can be found in biological tissues [1,2] and engineering applications [3–5], which have aroused our interest in such radiative heat transfer problems. Since the radiation streams in curved paths due to the spatial variation of the refractive index, the analysis of radiative heat transfer in a refractive medium is more difficult than that in a uniform index medium. Differential approximation methods [2–3,6–7] and numerous rigorous methods, including the discrete ordinate method (DOM) and its variations [8–11], the Monte Carlo method (MCM) [6,12–13], and the integral equation method (IEM) [14–16], have been developed to solve such kind of problems. Since the unknown intensity depends on position and direction, the DOM requires large amount of computer memory in higher dimensions. Furthermore, the MCM is a computation-intense method [11]. The IEM essentially transforms the integro-differential form of the radiative transfer equation in terms of intensity into a set of algebraic equations in terms of intensity moments with respect to directional cosine. Since the intensity moments are independent of direction, solving the IEM presents a better balance between computation cost and result accuracy. Wu and Hou derived the integral equations of intensity moments for radiative heat transfer in one- and two-dimensional (2-d) refractive media with black boundaries and applied the Nyström method to solve the resulting integral equations [14,15]. Degheidy et al. showed that the Galerkin method generated accurate results for radiative heat transfer in finite slabs with VRI [16].

Numerical solutions to the integral equations may be obtained by a variety of methods [17]. Some of the methods can be used to solve higher-dimensional integral equations for radiative heat transfer [15,18–21]. Computational complexity of mathematical operations is the main difficulty for solving higher-dimensional integral equations. The Galerkin method using inner product property offers highly accurate results, albeit at the price of quite laborious integrations [22]. A relatively simpler method is the collocation method with approximating the unknown variables by interpolation functions. In this work, we apply the collocation method with approximating the unknown variables by radial basis functions (RBFs) to solve two- and three-dimensional (3-d) integral equations for radiative heat transfer in refractive and participating media. RBFs were introduced by Hardy [23] and they form a useful tool for multivariate interpolation. They are also receiving increased attention for solving partial differential equation [24], integro-differential equation [25] and integral equations [26,27].

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Nomenclature

a_1	coefficient of the linear anisotropic phase function [–]
a_{kj}	coefficient of the phase function [–]
$\mathbf{a}_i$	coefficient vector of parametric equations [m]
$\mathbf{a}_{kj}(\mathbf{r})$	vector of functions, defined in Eq. (28a) [–]
$\mathbf{a}^*_{11}(\mathbf{r})$	vector of functions, defined in Eq. (29a) [–]
b_i	coefficient of parametric equations [m]
$\mathbf{b}_{kj}(\mathbf{r})$	vector of functions, defined in Eq. (28b) [–]
$\mathbf{b}^*_{11}(\mathbf{r})$	vector of functions, defined in Eq. (29b) [–]
$\mathbf{c}_{kj}(\mathbf{r})$	vector of functions, defined in Eq. (28c) [–]
$\mathbf{c}^*_{11}(\mathbf{r})$	vector of functions, defined in Eq. (29c) [–]
d_i	distance between $\mathbf{r}$ and $\mathbf{r}_i$ [m]
$\mathbf{d}_{kj}(\mathbf{r})$	vector of functions, defined in Eq. (28d) [–]
$\mathbf{d}^*_{11}(\mathbf{r})$	vector of functions, defined in Eq. (29d) [–]
$e_{kj}(\mathbf{r})$	function, defined in Eq. (28e) [Wm^{-2}]
$e^*_{11}(\mathbf{r})$	function, defined in Eq. (29e) [Wm^{-2}]
f	function, defined in Eq. (23) [Wm^{-2}]
I	radiative intensity [$\mathrm{Wm}^{-2}\ \mathrm{sr}^{-1}$]
J	radiosity [Wm^{-2}]
K	order of the phase function [–]
L	length of a cubic medium [m]
m_i	undetermined coefficient [Wm^{-2}]
$\mathbf{m}_{kj}$	vector of undetermined coefficients [Wm^{-2}]
$\mathbf{m}^*_{11}$	vector of undetermined coefficients [Wm^{-2}]
M_{kj}	intensity moment, defined in Eq. (5a) [Wm^{-2}]
M^*_{kj}	intensity moment, defined in Eq. (5b) [Wm^{-2}]
n	refractive index [–]
$\hat{\mathbf{n}}_w$	unit normal vector pointing into the medium [–]
N_r	number of data points per unit area for 2-d cases [m^{-2}]
p	scattering phase function [–]
P_k^j	associated Legendre function [–]
$\mathbf{r}$	position vector related to original point [m]
$\mathbf{r}^*$	position vector, defined in Eq. (15) [m]
R_s	radius of the outer semicircle [m]
s	curvilinear abscissa or path length [m]
s^*	path length, defined in Eq. (16) [m]
S	source function [$\mathrm{Wm}^{-3}\ \mathrm{sr}^{-1}$]
t	variable defined by $dt = ds/n$ [m]
T	local temperature [K]
$\mathbf{T}$	optical ray vector [–]
w	weight coefficient [–]
$\mathbf{y}_{RBF-IEM}$	vector of the quantities to be compared [–]
$\mathbf{y}_{\mathrm{benchmark}}$	vector of benchmark solutions [–]

Greek symbols

β	extinction coefficient [m^{-1}]
δ_{0j}	Kronecker delta [–]
ε	parameter for controlling the shape of RBFs [m^{-1}]
ε_w	emissivity of the wall [–]
ς	variable, $\varsigma = d\tau/dt$ [m^{-1}]
θ	polar angle [rad]
ρ	reflectivity [–]
σ	Stefan-Boltzmann constant [$\mathrm{Wm}^{-2}\ \mathrm{K}^{-4}$]
μ	directional cosine, $\mu = \cos\theta$ [–]
τ	variable, defined in Eq. (17) [–]
τ_L	optical thickness, $\tau_L = \beta L$ [–]
τ_R	optical thickness, $\tau_R = \beta R_s$ [–]
φ	azimuthal angle [rad]
Φ	radial basis functions [–]
$\mathbf{\Phi}$	vector, defined in Eq. (26) [–]
ω	scattering albedo [–]
$\hat{\mathbf{\Omega}}$	unit vector into a given direction [–]
Ω	solid angle [sr]

Subscripts and superscripts

b	blackbody
i	initial position or index of data points
w	wall
$\overline{()}$	approximation of ()

Utilizing RBFs to solve a higher-dimensional equation is a method based upon the scattered data approximation that approximates a function without any regular mesh generation on the domain [28,29]. Such a meshless method is particularly attractive when the domain considered cannot be expressed as product domains of lower dimensions.

In this work, we describe a RBF-based integral equation method (RBF-IEM) which extends the capability of the IEM to analyze radiative heat transfer in 2-d and 3-d refractive media with regular or irregular geometry. The implementations of the present method are illustrated by applying the method to several numerical examples. To show that the present meshless method is readily applied to radiative heat transfer in refractive media with complex geometries, we analyze radiative heat transfer in 2-d irregularly-shaped refractive media with black and diffusely reflecting surfaces. To the knowledge of the authors, no research works have been carried out to analyze radiative heat transfer in 3-d refractive media rigorously. Thus, we also apply the present method to analyze radiative heat transfer in cubic refractive index media.

2. Analysis

2.1. Physical model and integral equations for radiative heat transfer in refractive media with diffusely reflecting boundary

For a gray participating medium with a continuously VRI, the radiation transfer equation can be expressed as [30]

$$n^2(s)\frac{d}{ds}\left[\frac{I(s,\hat{\mathbf{\Omega}})}{n^2(s)}\right] + \beta I(s,\hat{\mathbf{\Omega}}) = S(s,\hat{\mathbf{\Omega}}), \tag{1}$$

where n denotes the refractive index of the medium, I the radiative intensity at the curvilinear abscissa s of the curved trajectory, as shown in Fig. 1, $\hat{\mathbf{\Omega}}$ the unit vector into the direction of intensity, β the extinction coefficient, and S the source function defined as

$$S(s,\hat{\mathbf{\Omega}}) = \beta(1-\omega)I_b(s) + \frac{\beta\omega}{4\pi}\int_{4\pi} I(s,\hat{\mathbf{\Omega}}')p(\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}')d\hat{\mathbf{\Omega}}' \tag{2}$$

with ω denoting the scattering albedo and p denoting the scattering phase function. In general, the scattering phase function can be expressed in terms of $\hat{\mathbf{\Omega}}(\mu,\phi)$ and $\hat{\mathbf{\Omega}}'(\mu',\phi')$ as

$$p(\mu,\phi,\mu',\phi') = \sum_{j=0}^{K}\sum_{k=j}^{K}(2-\delta_{0j})a_{kj}P_k^j(\mu)P_k^j(\mu')\cos[j(\phi-\phi')], \tag{3}$$

where K is the order of scattering, μ the cosine of the polar angle θ, ϕ the azimuthal angle, δ_{0j} Kronecker delta, $a_{kj} = a_k\frac{(k-j)!}{(k+j)!}$ with $a_0 = 1$, $0 \leqslant j \leqslant K$, $j \leqslant k \leqslant K$, P_k^j the associated Legendre function. Substituting Eq. (3) into Eq. (2), we obtain the expression

$$S(s,\hat{\mathbf{\Omega}}) = \beta(1-\omega)I_b(s) + \frac{\beta\omega}{4\pi}\sum_{j=0}^{K}\sum_{k=j}^{K}(2-\delta_{0j})a_{kj}P_k^j(\mu) \times [\cos(j\phi)M_{kj}(s) + \sin(j\phi)M^*_{kj}], \tag{4}$$