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MHD mixed convective stagnation point flow along a vertical stretching sheet with heat source/sink

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ABSTRACT

Aim of the paper is to investigate the effects of heat generation/absorption on MHD mixed convective stagnation point flow along a vertical stretching sheet in the presence of external magnetic field. The governing boundary layer equations are formulated and transformed into nonlinear ordinary coupled differential equations using similarity transformation and numerical solution is obtained by using Runge-Kutta fourth order scheme with shooting technique. The effects of various physical parameters such as velocity ratio parameter, mixed convection parameter, Hartmann number, Prandtl number and heat source/sink on velocity and temperature distributions are presented through graphs and discussed numerically. The skin friction coefficientand Nusselt number at the sheet are derived, discussed numerically and their numerical values are presented through tables.

1. Introduction

MHD flow and heat transfer of viscous fluids over a continuous stretching surface has great importance because of its many applications in engineering processes such as geothermal energy extraction, purification of metal from non-metal enclosures, plasma studies, aerodynamic extrusions of plastic sheets etc. Stagnation point flow is relevant to the bodies in high speed flow. It is used to reduce drag, in designing of thrust bearings and transpiration cooling etc. Stagnation point flow was first analyzed by Hiemanz [1]. Hiemanz used similarity transformation to reduce the governing Navier-Stokes equation into ordinary differential equations of third order subject to two point boundary conditions. Later many researches considered magnetic field, heat source/sink, suction/injection etc. to enhance the properties of fluids in stagnation point flow. Gupta and Gupta [2] studied heat and mass transfer on a stretching sheet with suction or blowing. Mixed convection in stagnation flows adjacent to vertical surfaces was analyzed by Ramchandran et al. [3]. Hassanien and Gorla [4] investigated combined forced and free convection in stagnation flow of micropolar fluids. Andersson [5] discussed MHD viscoelastic fluid flow past a stretching sheet. Chiam [6] presented heat transfer in a variable conductivity in a stagnation point flow towards a stretching sheet. Stagnation point flow of a viscoelastic fluid towards a stretching surface was studied by Mahapatra and Gupta [7]. Abel et al. [8] discussed buoyancy force and thermal radiation effects in MHD boundary layer flow over continuously moving stretching surface. Ishak et al. [9] analyzed mixed convection boundary layers in the stagnation point flow towards a stretching vertical sheet. Unsteady mixed convection flow of a micropolar fluid near the stagnation point was discussed by Lok et al. [10]. Layek et al. [11] presented heat and mass transfer analysis for boundary layer stagnation point flow towards a heated porous stretching sheet with heat absorption/generation and suction/blowing. Ishak et al. [12] discussed dual solutions in mixed convection flow near a stagnation point on a vertical porous plate. An effect of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet was studied by Sharma and Singh [13]. Pal [14] analyzed heat and mass transfer in stagnation point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Alharbi et al. [15] investigated heat and mass transfer in MHD viscoelastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Bhattacharyya and Layek [16] presented effects of suction/blowing on steady boundary layer stagnation point flow and heat transfer. Makinde et al. [17] discussed buoyancy effects on MHD stagnation point flow and heat transfer along heated stretching/shrinking sheet. Singh and Sharma [18] investigated dual solution for heat and mass transfer in the boundary layer flow along a vertical isothermal reactive plate near stagnation point. Mixed convection

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stagnation point flow on vertical stretching sheet with external magnetic field was studied by Ali et al. [19]. Shen et al. [20] presented MHD mixed convection slip flow near a stagnation point on a nonlinearly vertical stretching sheet.

2. Mathematical formulation

Consider a two-dimensional steady laminar flow of a viscous incompressible fluid along a vertical stretching sheet placed in x direction and y axis is normal to the sheet. u and v are the velocity components in x and y directions respectively. $u = u_e(x) = ax$ is the free stream velocity and the velocity by which the sheet is stretching is $u = u_w(x) = cx$ where both a and c are positive constants. An external magnetic field H_0 is applied normal to the sheet in the presence of heat source/sink. The governing equations of continuity, momentum and energy are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}u + g\beta(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}(T - T_\infty), \tag{3}$$

where μ_e is the magnetic permeability, σ is the electrical conductivity, ρ is fluid density, T is fluid temperature, T_∞ is temperature of free stream and temperature of the sheet is $T_w = T_\infty + bx$, b is a constant, for heated surface $b > 0$ so that $T_w > T_\infty$ and for cooled surface $b < 0$ and $T_w < T_\infty$, β is the volumetric coefficient of thermal expansion, $\upsilon(= \mu/\rho)$ is the kinematic viscosity, g is acceleration due to gravity, k is the thermal conductivity and C_p is the specific heat at constant pressure.

The boundary conditions are

$$v = 0, u = u_w(x) = cx, T = T_w(x) = T_\infty + bx \ at \ y = 0,$$

$$u = u_e(x) = ax, T = T_\infty \ as \ y \to \infty; \tag{4}$$

Due to hydrostatic and magnetic pressure gradient the forces will be in equilibrium as given below

$$-\frac{1}{\rho}\frac{dp}{dx} = u_e\frac{du_e}{dx} + \frac{\sigma\mu_e^2 H_0^2}{\rho}u_e, \tag{5}$$

Hence, the equation of momentum becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} - \frac{\sigma\mu_e^2 H_0^2}{\rho}(u - u_e) + \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty). \tag{6}$$

3. Method of solution

In order to get solution of Eqs. (1)–(3) with boundary conditions (4), we use the following transformations and dimensionless quantities

$$\eta = \sqrt{\frac{a}{\upsilon}}\,y,\ \ \psi = x\sqrt{a\upsilon}f(\eta),\ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\ \text{and}\ u = \frac{\partial\psi}{\partial y},\ \ v = -\frac{\partial\psi}{\partial x} \tag{7}$$

into equations, so that equation of continuity is automatically satisfied. The equation of momentum and energy become

$$f''' + ff'' - (f')^2 + 1 + Ha^2(1 - f') + \lambda\theta = 0, \tag{8}$$

$$\theta'' + Pr(f\theta' - f'\theta + \delta\theta) = 0, \tag{9}$$

where prime denotes the derivative with respect to η, $Ha(= \mu_e H_0\sqrt{\frac{\sigma}{\rho a}})$ is the Hartmann number, $\lambda(= \frac{Gr_x}{Re_x^2})$ is mixed convection parameter, $Gr_x(= g\beta(T_w - T_\infty)\frac{x^3}{\upsilon^2})$ is local Grashof number, $Re_x(= u_e(x)\frac{x}{\upsilon})$ is local Reynolds number, $Pr(= \frac{\upsilon}{\alpha})$ is the Prandtl number, $\delta(= \frac{Q}{\rho a C_p})$ is heat generation/absorption coefficient and the corresponding boundary conditions are reduced to

$$f = 0, f' = c/a = A,\ \ \theta = 1\ at\ \eta = 0$$

$$f' = 1, \theta = 0\ as\ \eta \to \infty \tag{10}$$

where $A(= \frac{c}{a})$ is the velocity ratio parameter.

Skin friction coefficient and Nusselt number are given by

$$C_f = \frac{2\tau_w}{\rho u_e^2} = Re_x^{-1/2}f''(0), Nu_x = \frac{xq_w}{k(T_w - T_\infty)} = -Re_x^{1/2}\theta'(0) \tag{11}$$

where the wall shear stress τ_w and the heat flux q_w, respectively are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ \ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Since equations (8) and (9) are highly nonlinear, it is difficult to find the closed form solutions. These equations are transformed into system of first order differential equations by substituting

$$f = f_1, f' = f_2, f'' = f_3, f''' = f_3', \theta = f_4, \theta' = f_5, \theta'' = f_5'.$$

Hence, the system of equations becomes

$$f_1' = f_2, f_2' = f_3, f_3' = f_2^2 - f_1 f_3 - 1 + Ha^2(f_2 - 1) - \lambda f_4$$
$$f_4' = f_5, f_5' = Pr(f_2 f_4 - f_1 f_5 - \delta f_4).$$

Subject to the following conditions

$f_1(0) = 0, f_2(0) = A, f_3(0) = s_1, f_4(0) = 1, f_5(0) = s_2$ and $f_2(\infty) = 1, f_4(\infty) = 0.$

Now Runge-Kutta fourth order method with shooting technique is used for step by step integration and calculations are carried out on MATLAB software.

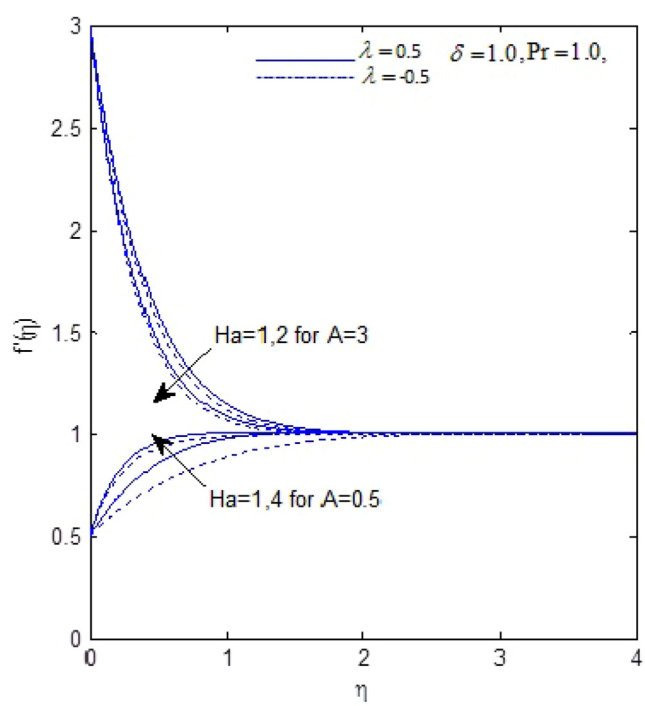

Fig. 1. Velocity profiles versus η for different values of Ha.