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About one splitting scheme for the nonlinear problem of thermal convection

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ABSTRACT

The paper is devoted to the questions of mathematical modeling of heating processes in hydrodynamics, namely to development and to mathematical justification of numerical algorithms for solving three-dimensional equations of free convection in natural variables. The purpose is to research implicit iterative schemes for the numerical solution of Boussinesq-type fixed (stationary) equations.
The research uses mathematical modeling, mathematical programming, the Visual Fortran programming language, and the Axum 7.0 graphics program. Computational mathematics and functional analysis methods are used for the mathematical argumentation of iterative algorithms.

The questions of convergence and estimate of a degree of convergence of one nonlinear splitting algorithm are considered, made for the difference analogues of the system of free convection steady-state equations in variables "velocity vector and pressure", written to shifted grids with symmetric approximation. The implicit iterative splitting algorithms for the difference analogues of the system of free convection steady-state equations in variables "velocity vector and pressure" are considered, written to shifted grids with symmetric approximation. The problems of stability of the difference problems according to the initial data and the right member, convergence and estimate of the linear algorithm degree of convergence were studied.

The results of this research can be useful in studies on difference schemes for hydrodynamic equations; they can also be used to further develop the theory of numerical solution of mathematical physics problems. The research results may be used in information system development for the automation of heat-aggregation exchange problem solving and as a teaching material for students learning mathematics, mechanics, and IT technologies.

1. Introduction

Development of theoretical and experimental researches of heat-process, connected with problems of atomic heat-engineering, chemical technology, construction technology, metallurgy, and also with ecology, lead to separation of free convection to independent section of liquid and gas mechanics. Currently, there are different mathematical models of free convection. The simplest one is the equation system in Boussinesq approximation consisting of Navier–Stokes equations and thermal conduction equations.

Nowadays due to the development of computational and informational technologies the opportunities of numerical solving of continuous area mechanics were opened, including problems of hydrodynamics on computer by means of final differences. Because of this the questions of numerical equation-solutions of heat convection draw the great attention of mathematicians and mechanics. At the same time methodological aspects of numerical modeling on the basis of heat convection equations are very complex and are not fully developed.

The mechanisms of free-convection flows determine various processes that have cognitive importance and wide applications. Free convection, occurring when heated side of a simply connected closed rectangular cavity was investigated by many authors [1–3]. As a result of these works the existence of several qualitatively different flow and transmission regimes was established, approximate boundaries of different modes of convection was identified, fields of temperature and velocity for each species of convective motion were investigated.

The coupling of two basic mechanisms of gravity-driven convection is called convective interaction. The classification of the applications of interaction mechanisms with different orientation of input (output) heat flux to the gravity force and certain convective interaction features with change of orientation for a model of differently heated squares have been shown [4].

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Paper [5] shows the results of a mathematical simulation of mixed convection in a low-temperature storage of liquefied natural gas with regenerative cooling. The regimes of mixed convection in a closed area with various arrangements of the input and output sections of the masses were investigated. The two-dimensional nonstationary problem in the Navier–Stokes model with "vorticity – stream function – temperature" dimensionless variables was examined.

Detailed numerical simulations were carried out for transient laminar opposing mixed convection in a rectangular inclined channel with both walls suddenly exposed to discrete isothermal flush-mounted heat sources simulating electronic components. Using the vorticity-stream function formulation of the unsteady two-dimensional Navier–Stokes and energy equations, the governing equations are solved numerically using the control volume method [6].

Free convection of ferrofluid in a cavity heated from below in the presence of an external magnetic field is studied numerically using the Lattice Boltzmann method [7]. The enclosure is filled with a mixture of kerosene–cobalt. This investigation was compared with other experimental and numerical works and found to be in excellent agreement. The effect of the Rayleigh number, magnetic coefficient, heat source length, and volume fraction of cobalt on the flow and heat transfer characteristics were examined. Results show that particles with a smaller size have a better ability to dissipate heat; a larger volume fraction would provide a stronger driving force, which leads to an increase in the temperature profile. The Nusselt number has a direct relationship with the Rayleigh number and heat source length and a reverse relationship with size of the nanoparticle and volume fraction of cobalt.

A very effective and higher order numerical scheme – Control Volume-based Finite Element Method (CVFEM) – is used to solve the resulting coupled equations. The numerical investigation is carried out for different governing parameters, namely: the Rayleigh number, nanoparticle volume fraction, and inclined angle of elliptic inner cylinder. The effective thermal conductivity and viscosity of nanofluid are calculated using the Maxwell–Garnetts (MG) and Brinkman models, respectively [8].

In the works of Gnevanov and Tarunin [9] the idea of the stability of convective equilibrium and the movements of the liquid at the presence of non-uniform heat source in a flat layer of arbitrary orientation was considered.

Stationary solutions of a system coupling singular Navier–Stokes equations to a enthalpy-heat equation are described in [10]. This system may model the solidification process of certain classes of materials by taking into consideration the possibility of flow in the melt; thus, the singular Navier–Stokes equation only holds in the a priori unknown molten region and one has a free-boundary value problem.

Research [11] described a new algorithm for the numerical solution of the Navier–Stokes equations coupled with the convection–diffusion equation.

In studies devoted to the construction of new models of convection and the investigation of exact solutions (see, e.g., [12]), great attention is paid to the estimation of the effect of boundary conditions and various factors. The main difficulties of boundary problem solving for Boussinesq-type equation system are associated with nonlinear convective summands with regard to the transfer occurrence, which not always allows obtaining an exact solution.

In [13], Consiglieri, Nečasová, and Sokolowski proved the existence, uniqueness, and regularity of weak solutions to the Maxwell–Boussinesq approximation to the Fourier–Navier–Stokes flows under an electromagnetic field. The authors studied the thermoelectromagnetic flow model with temperature-dependent material coefficients W1,p -regularity ($p > 2$) is obtained for the weak solutions of the problem in three-dimensional smooth domains with mixed (Dirichlet–Robin) boundary conditions for temperature, the Dirichlet boundary condition for velocity, and mixed (Dirichlet–Neumann) boundary conditions for the electric potential.

A generalization of the Ostroumov–Birikh solution to free convection equations, which describes a two-layer flow in the inclined plane, was investigated. The lower boundary of the system is a fixed rigid wall, on which a constant temperature gradient is maintained and the upper boundary is the free surface. The effect of the problem parameters on the flow characteristics was analyzed and the problem of stability of these flows with respect to longwave perturbations was studied. The Navier–Stokes equations in the Oberbeck–Bussinesq approximation are used as the mathematical model [14].

The finite-difference method allows solving the problems of the non-stationary convection theory effectively. For instance, Tarunin [15] solved the problem of a flat convective heating fluid in a square cavity, on the borders of which the temperature was suddenly changed and further constant temperature was maintained.

Tarunin's research [15] gives detailed data about convection investigations in a rectangular area during one-sided heating, where one of the disadvantages of such investigations is the calculation absence of three-dimensional real courses, which provides more detailed results in two-dimensional cases. Two-dimensional and three-dimensional equations differ not only in terms of space; they essentially differ in approaches to solving these equations.

Numerical algorithms in the "whirlwind of speed, current function" variables are commonly used to solve two-dimensional equations of hydrodynamics and heat-convection in closed areas [2,16,17]. The main advantage of this approach is that it is possible to reduce the number of unknowns, i.e. instead of three equations of uncompressible liquid in "speed, pressure" variables, two equations of relative whirlwind and current function are examined. In the three-dimensional case, the introduction of the whirlwinding vector and potential vector increases the number of equations and unknowns. This is why algorithms developed on the basis of equations in "velocity vector, pressure" variables are widely used for the numerical solution of three-dimensional equations of uncompressible liquid [18–20].

Papers [21–24] are devoted to the numerical study of thermal convection differential problems described by Eqs. (8)–(10). They mostly consider the numerical implementation of developed iterative schemes without a mathematical substantiation of the applicability of the used algorithm.

The numerical study of convection by finite difference method was covered in research [25], which dealt with different problems requiring essentially nonlinear interpretation: the structure of the stationary convective motions in a closed cavity with large temperature differences [26], nonlinear secondary flows arising from the crisis, the stability of the equilibrium or of the stationary motion [27,28], and supercritical motion arising in the presence of a periodic modulation of parameters [29].

In work [30] for two-dimensional Stokes and Navier–Stokes equations recorded in terms of the stream function – a whirlwind initial boundary value problem in a closed area is considered. Difference boundary problems are formulated on the basis of the difference Crank–Nicolson scheme for the function of the vortex and the fourth-order approximation scheme for stream function. For the numerical solution of differential Stokes equations the Fourier method in conjunction with the modified method of pivotal condensation is used. In the case of Navier–Stokes equations for calculating function vortex splitting by physical processes (convection and diffusion) is used. For linear and difference equations estimates of the solution are obtained. As well as the stability of initial boundary value problem is proved.

Despite tremendous advances made in the last few decades in the field of computational fluid dynamics, numerical schemes for