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Equilibrium and non-equilibrium thermodynamic analysis of high-order dual-phase-lag heat conduction

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ABSTRACT

The compatibility of dual-phase-lag (DPL) heat conduction with the hypothesis of local thermodynamic equilibrium (LTE) and extended irreversible thermodynamic is investigated. The analysis is based on evaluating the entropy production rate (EPR) for a solid slab that is exposed to a sudden temperature gradient at its boundaries. In this regard, an analytical solution is presented for the first-order and second-order approximations of the DPL model. It is shown that by extending the non-Fourier Cattaneo–Vernotte (C–V) wave model to the high-order DPL model, the unphysical values of the slab temperature that exist in the solution of C–V wave model cannot be generally eliminated. However, a set of phase lag times for the DPL model exist based on which these unphysical values can be omitted by increasing the diffusion versus the wavelike features.

1. Introduction

It is well known that the wavelike behavior of heat conduction becomes dominant in the heat transfer processes including low temperatures [1], inhomogeneous inner structure materials [2,3], high heat flux [4], short pulse laser heating [5], and in temporal and spatial micro- and nano-scales [6]. Although the Cattaneo–Vernotte (C–V) wave model has been widely utilized in the aforementioned applications [1–6], the second law analysis of this model indicates its incompatibility with the theory of local thermodynamic equilibrium (LTE) [7–9]. In fact, it is shown that the C–V wave model may result in both positive and negative values for entropy production rate (EPR) that is in contrast with the Clausius inequality.

In this regard, the theories of extended irreversible thermodynamics are proposed based on which the C–V wave model yields only positive EPR values [10]. However, some of these theories are inconsistent with each other or cannot be verified since they are not validated experimentally [8]. Therefore, Barletta and Zanchini [8] obtained a domain in which the hyperbolic energy equation resulted from the C–V wave model is compatible with the LTE. It is indicated that the results of C–V wave model in this domain differ from those from the Fourier law [8].

The dual-phase-lag (DPL) constitutive equation, that is proposed by Tzou [6], is another non-Fourier heat conduction model that has both wavelike and diffuse behaviors in a heat conduction process, simultaneously. As stated by Antaki [11], the DPL model has attracted many researchers, because it is supported experimentally [12]. It should be noted that the DPL model can reflect the microscopic and macroscopic responses in both time and space scales [6]. In other words, based on the values of phase lag times in this model, the diffusion feature may overcome the wavelike feature or vice versa. When the diffusion feature is prevailing, the DPL model shows over-diffused behavior in heat conduction processes. Hence, investigation on the consistency of the DPL model with LTE or extended irreversible thermodynamic can be useful for future judgment about where it is applicable. In a recent study by Askarizadeh and Ahmadikia [9] a set of phase lag times has been obtained for the DPL model that makes it compatible with the LTE hypothesis.

In this regard, Jou and Sancho [13] presented a finite initial value for the temperature gradient based on the thermodynamic stability analysis to restrict the temperature overshooting in the DPL heat conduction. Al-Nimr et al. [14] studied both the equilibrium and non-equilibrium EPR of the DPL heat conduction process in a semi-infinite medium. It is concluded that the hypothesis of LTE cannot generally describe the DPL heat conduction process in such medium that undergoes a constant surface temperature. Serdyukov [15] obtained the DPL model of transport equation based on a new postulate of extended irreversible thermodynamics and

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Nomenclature

k	thermal conductivity
L	half thickness of the slab
p	Laplace domain parameter
q	heat flux
r	spatial coordinate vector
s	entropy per unit mass
t	time
T	temperature
u	energy per unit mass
x	spatial coordinate

Greek symbols

α	thermal diffusivity
β	a function in Laplace domain
Γ	dimensionless temperature gradient relaxation time
η	dimensionless time
θ	dimensionless temperature
λ_n	eigenvalues
Λ	dimensionless heat flux relaxation time
ξ	dimensionless coordinate
φ	dimensionless entropy production rate
χ	dimensionless heat flux
υ	specific volume
ρ	density
σ	entropy production rate
τ	relaxation time

Subscripts

0	initial conditions
T	temperature
q	heat flux
w	wall

Superscripts

$*$	non-equilibrium state
$-$	transformed function in Laplace domain

the assumption that entropy is a function of internal energy and its time derivatives. Vadasz [16] demonstrated that the lack of local thermal equilibrium may lead to occurrence of thermal waves and resonance according to the approximate equivalency between the DPL and Fourier heat conduction models in porous media. In order to guarantee the stability of the solutions of the DPL heat conduction equation, a range of temperature gradient and heat flux relaxation times is analyzed for different kinds of approximations in the DPL theory [17]. Xu [18] developed two types of extended irreversible thermodynamic theories to prevent the possible violation of the thermodynamics second law by the DPL model due to the thermal vibration phenomenon under the LTE assumption. The non-equilibrium heat and mass transfer during the ultrafast response has been investigated based on the lagging behavior concept by Tzou and Xu [19] and the necessity of considering the thermal lagging in transfer processes at micro-scale is shown in the framework of the non-equilibrium thermodynamics. Rukolaine [20] described the first-order approximation of the DPL model by the Jeffreys-type equation. It is solved as the governing equation for an initial value three-dimensional problem with a positive localized source of short duration. Based on the resulted negative values, it is concluded that the Jeffreys-type equation of the DPL model is not an appropriate model for heat conduction. Fabrizio and Lazzari [21] analyzed the stability of the DPL constitutive equations based on the second law of thermodynamics.

Furthermore, several numerical and conceptual studies have been presented based on the high-order DPL model of heat conduction that implies its applicability and stability in engineering problems [22–25]. Quintanilla and Racke [22] investigated the qualitative aspects in the DPL heat conduction and concluded that whenever both phase lag times are positive, the problem in the framework of the DPL model becomes well-posed. The first-order and second-order DPL heat conduction are analyzed in bioheat transfer problems through skin tissue under surface heating boundary conditions by Xu et al. [23]. It is concluded that the non-Fourier aspect is important and should be considered in thermal analysis of skin tissue. Single phase and DPL thermomechanical models have been studied to show their applicability in heat conduction processes by Dreher et al. [24]. Tissue surface temperature and thermal lagging effect in tissue-mimics during laser irradiation are evaluated based on the DPL model to indicate that the heat conduction process in biological tissues can be well approximated utilizing the non-Fourier models [25]. Liu [26] analyzed the non-equilibrium heat transport in biological tissues by employing the high-order approximation of the DPL model based on a hybrid analytical-numerical method. It is demonstrated that the high-order terms enhance the non-Fourier feature of heat transfer.

Due to the wide applications of the DPL model in modern heat transfer engineering problems, this research extends the previous study of Barletta and Zanchini [8] by utilizing a unified analytical approach based on Laplace transform method and inversion theorem in complex plane for both the first-order and second-order DPL models. In this regard, the results of hyperbolic heat conduction in a plane slab, that is subjected to a sudden temperature change at its boundaries [8], are developed to investigate the effects of high-order terms in the DPL heat conduction and also the effect of temperature gradient relaxation time on the equilibrium and non-equilibrium EPRs. On the other hand, the gradient precedence (GP) heat flow regime in the DPL model, that occurs when the temperature gradient relaxation time (τ_T) is less than heat flux lag time (τ_q), and the flux precedence (FP) regime ($\tau_T > \tau_q$) are analyzed by the first-order and second-order DPL models. In addition, this paper presents an analytical solution for the second-order DPL model of conduction heat transfer equation that has not been reported in the literature so far.

2. Mathematical formulation

The parabolic and hyperbolic conduction heat transfer equations can be derived by applying the Fourier and C–V wave models to the following local energy balance equation for a solid slab, respectively. It is assumed that no heat generation exists and the thermo-physical properties are constant.

$$\nabla \cdot \boldsymbol{q} + \rho \frac{\partial u}{\partial t} = 0 \tag{1}$$

A combination of the parabolic and hyperbolic features in heat transfer processes can be obtained by employing the following DPL model.

$$\boldsymbol{q}(\boldsymbol{r}, t + \tau_q) = -k \nabla T(\boldsymbol{r}, t + \tau_q) \tag{2}$$