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Absolute instability: A toy model and an application to the Rayleigh–Bénard problem with horizontal flow in porous media

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ABSTRACT

The concept of absolute instability is surveyed and applied to the study of the Rayleigh–Bénard problem in a horizontal porous layer with longitudinal flow. The survey is aimed to provide a simple introduction to absolute instability by employing a toy model based on a one-dimensional Burgers' equation. The method of analysis is based on the steepest descent approximation, for large times, of the Fourier integral expressing the wavepacket perturbation of the basic solution. The analysis of Burgers' equation is a suitable arena for the illustration of the elementary features of absolute instability. Then, the onset of absolute instability in a horizontal porous layer with a prescribed wall temperature difference between the boundaries and subject to a longitudinal flow is analysed. The seepage flow is modelled through Darcy's law by assuming a finite Darcy–Prandtl number. It is shown that the transition from convective to absolute instability occurs at supercritical conditions, except for the limiting case when the horizontal flow rate is vanishingly small. In this special case, corresponding to the Darcy–Bénard problem, the condition of convective instability yields also absolute instability. The effects of the governing parameters, the Péclet number and the Darcy–Prandtl number, on the onset of absolute instability are studied.

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1. Introduction

Linear instability of a basic stationary flow in fluid mechanics is investigated by testing the response of the basic flow to normal modes of perturbation with a vanishingly small amplitude. The resulting eigenvalue analysis yields a neutral threshold to instability. In the simplest cases, this threshold can be represented as a neutral stability curve drawn in the two-dimensional (k, R) space, where k is the wave number and R is the parameter driving the transition to instability. In different sample cases, R can be either the Reynolds number, the Rayleigh number, or the Marangoni number. The point along the neutral stability curve where R is minimum yields simultaneously the critical wavenumber, k_c, and the critical parameter, R_c. A stable response to a normal mode of perturbation occurs when $R < R_c$, while instability is for $R > R_c$. Actually, normal modes of perturbation are just the Fourier components of a disturbance, represented as a wavepacket, acting on the basic stationary flow. The wavepacket is expressed as a Fourier integral over all possible normal modes with different wavenumbers. It is clear that supercritical regime $(R > R_c)$ is a necessary condition for an unstable evolution (growth) of the wavepacket at a fixed position and for large times, but this condition is generally not sufficient. A wavepacket may be damped in time at any fixed position, for large times, even if $R > R_c$. This may be the case when the basic stationary flow rate in a given direction is nonzero. In this case, growing wavepacket perturbations may be convected away before their growth for large times can be recorded by an observer standing at a fixed position. The regime where perturbation wavepackets grow for large times at any given position is called absolute instability. Here, the adjective "absolute" is meant to mark a distinction with respect to the supercritical regime $(R > R_c)$, usually called convective instability. If convective instability means $R > R_c$, absolute instability means $R > R_a$ where R_a is not less than R_c.

The concept of absolute instability was developed more than fifty years ago in the area of plasma physics [1–3], but its interest for fluid mechanics was soon recognised and reported in classical textbooks such as Landau and Lifshitz [4], Drazin and Reid [5], or the more recent Schmid and Henningson [6]. Several papers on the absolute instability of fluid flows have been published in the past decades surveying the main physical and mathematical aspects of this topic. Among the many, we mention Huerre and Monkewitz [7,8], Carrière and Monkewitz [9], Suslov [10], and Juniper et al. [11].

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Nomenclature

a	small distance in the complex k plane, Eq. (25)
A, B	constants, Eq. (61)
c_a	acceleration coefficient
$\mathcal{C}, \mathcal{C}^*$	paths in the complex k plane
C	constant amplitude
$\mathbb{C}, \mathbb{R}$	set of complex numbers, set of real numbers
$\mathcal{D}$	differential operator, Eq. (39)
$\mathcal{D}_L$	linear differential operator, Eq. (41)
$F(t)$	function of time, Eq. (11)
$\mathbf{g}$	gravity acceleration
g	modulus of $\mathbf{g}$
H	channel height
i	imaginary unit
k	wavenumber
k_0	saddle point
k_1	constant wavenumber
K	permeability
m	integer, Eq. (24)
n	integer, Eq. (61)
p	positive integer, Eq. (21)
$\mathrm{P}_{\pm}$	solutions of Eq. (68) given by Eq. (70)
Pe	Péclet number, Eq. (54)
Pr	Darcy–Prandtl number, Eq. (48)
r	polar coordinate in the k plane, Eq. (20)
R	parameter driving the transition to instability
Ra	Darcy–Rayleigh number, Eq. (50)
$\mathfrak{Re}, \mathfrak{Im}$	real part, imaginary part
$S(x)$	source term, Eq. (39)
t	time, Eq. (47)
T	temperature, Eq. (47)
T_0	reference temperature
u, v	x and y components of velocity, Eq. (47)
U_0	constant horizontal channel velocity
w	perturbation of W, Eqs. (2) and (40)
W	solution of Burgers' equation, Eq. (1)
W_0	basic solution of either Eq. (1) or Eq. (39)
x, y, z	Cartesian coordinates, Eq. (47)

Greek symbols

β	thermal expansion coefficient
γ	dimensionless parameter, Eq. (48)
Γ	Euler's gamma function
δ	Dirac's delta function
δk	infinitesimal wavenumber increment
ΔT	reference temperature difference
ε	perturbation parameter, Eqs. (2) and (55)
θ	argument of $\lambda^{(p)}(k_0)$
Θ	temperature perturbation, Eq. (55)
$\varkappa$	average thermal diffusivity
λ	complex growth rate, Eqs. (5), (42) and (58)
ν	kinematic viscosity
ξ	volumetric heat capacity ratio
φ	polar coordinate in the k plane, Eq. (20)
$\phi(k)$	function of k, Eq. (11)
ψ	streamfunction perturbation, Eq. (55)
Ψ	streamfunction, Eq. (52)
Ψ_0, T_0	basic solution, Eq. (54)
ω	angular frequency, $\mathfrak{Im}\{\lambda\}$

Superscripts, subscripts

$\hat{}$	Fourier transform
$\bar{}$	complex conjugate
a	threshold to absolute instability
c	critical value
r, i	real part, imaginary part

The analysis of the transition from convective to absolute instability with respect to seepage flows in porous media is relatively recent and the literature is not abundant [12–21]. The description of the methods to investigate the transition to absolute instability presents conceptual and mathematical difficulties that may discourage most of the people actively engaged in the research on the instability in porous media. This paper aims to clarify and, wherever possible, simplify these methods. One of the elements that can induce some confusion is the link between analysis of absolute instability and use of spatially growing and time-periodic normal modes, also called spatial modes. The use of such modes is perfectly justified in special studies dealing with the streamwise diffusion of time-periodic oscillations imposed at a fixed position, say the inlet section of a given flow. However, spatial modes are not strictly necessary in the study of transition to absolute instability, as it will become clear in the forthcoming sections. Our approach to absolute instability is entirely based on the steepest descent approximation, which describes the large time behaviour of a perturbation wavepacket. The basic concepts and methods are illustrated by employing a toy model of unstable flow, based on Burgers' equation. Then, the transition to absolute instability is analysed for a porous medium flow subject to a vertical temperature gradient. The seepage flow, described by Darcy's law with a finite Darcy–Prandtl number, turns out to display a convective instability that has been analysed by Dodgson and Rees [22]. We will show that the instability may turn from convective to absolute under supercritical conditions, whenever the Péclet number associated with the basic flow is nonzero. This study is based on an analytical dispersion relation, so that the numerical work just relies on simple algorithms for root finding, such as the Newton–Raphson method.

2. A simple example: Burgers' equation

A very simple example to start illustrating the concepts of convective instability and absolute instability is given by the one-dimensional Burgers' equation,

$$\frac{\partial W}{\partial t} + W\,\frac{\partial W}{\partial x} = \frac{\partial^2 W}{\partial x^2} + R(W - W_0), \tag{1}$$

where $R \in \mathbb{R}$ and $W_0 \in \mathbb{R}$, $\mathbb{R}$ being the set of real numbers. Evidently $W = W_0$ is a basic solution of Eq. (1).

The choice of Burgers' equation as a toy model for the illustration of the method is made for its pedagogical value. This equation is a prototypical convection–diffusion equation that mimics some of the basic physical features of a fluid flow system. Despite its poor connection to the real-world fluid flows, with Burgers' equation the concept of absolute instability can be illustrated quite effectively without the distraction of mathematical complexities.

2.1. Stability analysis

A linear stability analysis of the constant solution, $W = W_0$, is carried out by perturbing it, namely by substituting

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