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Acoustic wave effect on a stability of convective flow in a horizontal channel subjected to the horizontal temperature gradient

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ABSTRACT

The effect of acoustic wave propagating in longitudinal direction on stationary convective flow in a horizontal channel of rectangular cross-section in the presence of horizontal temperature gradient and its stability is studied. It is shown that in the presence of sufficiently intensive acoustic wave stationary flow is of coaxial character, the fluid moves in the direction of temperature gradient near sidewalls and in the opposite direction in the interior of the channel. At $Pr < \sim 0.2$ hydrodynamic shear instability mode is most dangerous. This mode is strongly stabilized by acoustic wave at low Prandtl numbers. The range of Prandtl number values where acoustic wave can stabilize the basic flow becomes wider with the channel width growth. At high Prandtl number values the spiral instability modes are most dangerous. The oscillatory spiral mode is slightly stabilized by acoustics. The monotonic spiral mode is destabilized by acoustic wave, especially in the wide channels. In general, for all instability modes the sidewalls suppress the development of perturbations and stationary flow in the channel is more stable than that in plane horizontal layer. In the channel of square cross-section the oscillatory spiral mode is completely suppressed.

1. Introduction

The problem of thermal buoyancy convection in the presence of horizontal temperature gradient has multiple practical applications and is actively studied. Flows of such kind develop in some technological processes and geophysical phenomena. These are some types of motions in the ocean, crust and mantle of Earth, transportation processes in shallow water bodies, and motion of the melt in crystallization apparatus in horizontal variant of directional solidification method. In the latter case heat and mass transfer processes type considerably influence the microstructure of grown crystal (see [1]). That as why the understanding of physical mechanisms determining melt dynamics and its stability is important. Often the suppression of primary instability leads to stabilization of the process in general, which is very desirable.

In the literature there is a large number of works devoted to investigation of stationary convective flow structure in the presence of horizontal temperature gradient for different geometries (infinite horizontal layer [2], infinite horizontal channels of different cross-sections [3], horizontal channels of finite length [4–6], etc.)

The most studied is the problem of stability of convective flow in infinite horizontal layer with rigid conductive boundaries (detail survey is given in [2]). Due to the presence of longitudinal temperature gradient the reduction of the problem on three-dimensional perturbations of plane-parallel motion to two-dimensional one is not possible. Two extreme cases are possible: of two-dimensional perturbations as rolls perpendicular to basic flow (transversal rolls), and three-dimensional spiral perturbations as rolls with axes parallel to basic flow (longitudinal rolls). As analysis show, there are several mechanisms of instability. At low Prandtl numbers instability is related to hydrodynamic mechanism, for $Pr = 0$ critical Grashof number $Gr_c = 495$ coincides with the one for isothermal flow. This mode was first investigated analytically in [7], later it was studied numerically by Galerkin method in [8]. As Prandtl number grows, this mode is stabilized sharply, and for $0.14 < Pr < 0.44$ three-dimensional oscillatory mode discovered in [9] is the most dangerous. For high Pr the most dangerous is monotonous even three-dimensional mode (it was discovered in [6], later it was studied by methods of asymptotic analyses in [10] and numerically in [11]). There are also odd three-dimensional and two-dimensional modes of Rayleigh origin, but they do not becomes the most dangerous at any values of Pr.

In [12–14] stationary convective flow in long horizontal channel of rectangular cross-section with longitudinal temperature gradient applied to the boundary and its stability with respect to small

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perturbations were studied. In [12–13] the case of zero Prandtl number was investigated, the structure of basic flow for conductive and adiabatic boundaries was considered, the critical Grashof number values relative to three-dimensional hydrodynamic perturbations periodical along the channel axis were obtained using stability theory, transition to the limit case of infinite layer with the relative channel width growth was analyzed. In [14] the problem was considered for arbitrary Prandtl numbers. It was shown that while at zero Prandtl number the flow along channel axis is plane-parallel, at non-zero Pr plane-parallel solution does not exist, in the cross-section four vortices develop. At low Pr instability mechanism is the same as in the case of infinite layer, as Prandtl number grows steady hydrodynamic mode is strongly stabilized, and starting from some its value (depending on relative channel width) instability develops as vortices near sidewalls. With further Prandtl number growth the oscillatory three-dimensional mode becomes most dangerous at first, then steady three-dimensional mode of Rayleigh type. As channel width decreases all modes are stabilized.

In [15] stationary convective flow and its stability in the horizontal channel of rectangular cross-section with adiabatic boundaries in the presence of horizontal temperature gradient were studied numerically. Dependencies of critical Rayleigh number on Prandtl number at different ratios of sides of transversal cross-section of the channel are determined.

Numerical modeling of three-dimensional convective flows in horizontal channel of square cross-section with isothermal sidewalls was carried out in [16], dependencies of critical Grashof number, at which bifurcations to oscillatory mode take place, were obtained.

The presence of large number of instability mechanisms make possible application of different supplementary factors (boundary motion, stationary and rotating magnetic fields and other) to control stability of convective flow in the presence of horizontal gradient of temperature. Numerical modeling of convective flow of conductive fluid in elongated parallelepiped in the presence of stationary magnetic field was carried out in work [17], the effectiveness of the method for fluids like molten metal was demonstrated. The influence of stationary magnetic field on the stationary convective flow in horizontal channel of rectangular cross-section, elongated vertically, and its stability was studied numerically in [18]. At zero and low Prandtl number values a significant stabilizing action of vertical transversal magnetic field was founded, while horizontal transversal magnetic field with strength below some threshold on the contrary causes destabilization at $Pr = 0$ and low channel width. Analytical and numerical investigation of the influence of rotating magnetic field on convective flow in horizontal channel of circular cross-section in the presence of horizontal temperature gradient was carried out in [19], it was shown, that at low Prandtl numbers $Pr < Pr_{t,0} \approx 3 \cdot 10^{-4}$, before basic flow mode change, there is a strong stabilizing action of magnetic field. At high Pr weak stabilization is observed.

A perspective method is also the use of acoustic wave propagating along channel axis. The work [20] is devoted to the impact of acoustic influence on the formation and stability of convective flow in plane horizontal layer of fluid. Linear stability of convective flow with respect to two-dimensional, spiral and three-dimensional perturbations was studied. It was found that at low Prandtl numbers acoustic influence stabilizes convective flow. In the presence of acoustic influence a parameter range appears in which perturbations of inclined wave type are most dangerous. It was shown that all mentioned perturbation types develops as supercritical bifurcations, stability domains of two-dimensional and spiral secondary flows was founded.

The influence of ultrasound beam on the stability of convective flow in rectangular parallelepiped and in infinite layer was considered in [21] and [22]. The stabilizing action of ultrasound beam at low Prandtl numbers was founded; it was shown that in the case of parallelepiped of moderate length stabilization takes place as well, but in other domains of parameter values destabilization of basic flow is possible. The effect of acoustic wave on the process of directional solidification was investigated experimentally in [23,24]. The reduction of striation inhomogeneity of monocrystals grown by Chohralski method and reduction of convective flows was discovered.

Investigation of influence of thermal properties of boundaries on the stability of convective flow in an infinite plane layer in the presence of horizontal temperature gradient and acoustic wave propagating in the direction of this gradient was carried out in [25]. Different modes of instability were considered. It was found that at low Prandtl numbers, when two-dimensional or three-dimensional oscillatory modes are most dangerous, acoustic wave makes stabilizing effect. It was shown that purely acoustic flow (with $Gr = 0$) is absolutely stable to perturbations of longitudinal rolls type. Instability of purely acoustic flow to other types of perturbations was not found; probably it is stable relative to them as well.

This work is devoted to investigation of influence of acoustic wave on the stability of convective flow in long channel with conductive boundaries. This problem was not studied earlier.

2. Problem formulation

Let us consider the infinite horizontal channel of rectangular cross-section with rigid walls of height H and width L. Coordinate axis z is directed along the channel axis at the distance $H/2$ and $L/2$ from walls. Axis y is directed vertically upward, axis x correspondingly is directed horizontally perpendicularly to axis z. The temperature on the walls is directly proportional to longitudinal coordinate: $T = Az$, A is a constant. In the direction of temperature gradient (and axis z) acoustic wave is propagating, velocity of which have the form $\boldsymbol{w} = a\omega \boldsymbol{k} \cos(\omega(t - z/c))$ (where a is amplitude, ω is frequency, c is sound velocity, $\boldsymbol{k}$ is unit vector of z axis). We suppose that period of oscillations is small compared to characteristic viscous and thermal time and condition $H/\lambda \ll \beta\Delta T$, where λ is wavelength, β is thermal expansion coefficient, ΔT is characteristic temperature difference, is satisfied. This allows to use averaged description of fluid motion. Equations of thermoacoustic convection under above assumptions were obtained in [26]. Equations for averaged fields from [26] are similar to ordinary equations for thermal convection in Boussinesq approximation, and influence of acoustic wave, generating in boundary layers near rigid walls mean vorticity which propagates through the entire volume of fluid due to convection and viscosity, which leads to formation of acoustic flow, is described using effective boundary conditions on the external edge of boundary layer. These conditions can be translated directly to the rigid surface because boundary layer thickness is small. Analysis gives for the velocity on the channel walls the value $v_z = w = 3a^2\omega^2/(4c)$ [26,27].

Taking quantities H, ν/H, H^2/ν, AH and $\rho\nu^2/H^2$ as scales for length, velocity, time, temperature and pressure respectively, we obtain following system of equations:

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla p + \Delta \boldsymbol{v} + GrT\boldsymbol{\gamma}, \tag{1.1}$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{v} \cdot \nabla)T = \frac{1}{Pr}\Delta T, \tag{1.2}$$

$$\nabla \cdot \boldsymbol{v} = 0, \tag{1.3}$$