International Journal of Heat and Mass Transfer 104 (2017) 503–509

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

An analytical solution for two-dimensional modeling of repetitive long pulse laser heating material

Guibo Chen, Yaodong Wang, Jiajia Zhang, Juan Bi *

School of Science, Changchun University of Science and Technology, Changchun 130022, PR China

ARTICLE INFO

Article history:
Received 3 July 2016
Received in revised form 24 August 2016
Accepted 24 August 2016

Keywords:
Analytical solution
Laser heating
Repetitive long pulse
Conduction

ABSTRACT

Laser conduction limited heating finds wide application in surface processing industry. Modeling of the laser heating can give an insight into the physical processes. In this paper, repetitive long pulse laser heating of solid materials is considered, and an analytical solution for two-dimensional modeling of the temperature distribution is obtained using the method of separation of variables combining with Laplace transformation. It is found that the results obtained from the analytical solution agree well with the existing finite element method. Temperature distributions for different radial locations, axial locations, duty cycles and repetitive frequencies are calculated, and effects of these parameters on temperature distributions are analyzed. The results of this study can give some theoretical basis for revealing the mechanism of laser heating and provide some guidance for experiments of repetitive long pulse laser heating materials in the next work.

1. Introduction

Repetitive pulse laser heating material is widely used in laser processing or laser damage [1]. When the surface of material is irradiated by a repetitive pulse laser, the absorption of material to the incident laser is presented in an intermittent cumulation mode, so it has the energy accumulation effect and can provide sufficient energy deposited onto the surface, but the single pulse or continuous wave (CW) laser does not have it [2].

Mathematical modeling is an important method to find physical laws and characteristics of the laser heating. It is able to reduce the experimental cost and minimizes the experimentation time [2]. Moreover, modeling studies can provide results of laser heating for sufficient conditions even if in the environment that traditional experiment cannot achieve. The mathematical modeling of physics problems can be divided into numerical modeling and analytical modeling. Analytical modeling establishes a direct functional relation between the parameters and the laser heating process, which can provide very useful information for revealing mechanism of the heating and parameters optimization of the laser [3]. In recent years, there are some studies on the analytical modeling of repetitive pulse laser heating materials. Khenner et al. [4] obtained an analytical solution of the classical heat conduction problem for a solid film with a surface that is simply deformed and irradiated by repetitive laser pulses using the method of separation of variables. In their studies, convective heat losses from the surface and the film-substrate interface are taken into account. Kalyon and Yilbas [2] derived a closed form solution of the dimensionless temperature rise including the cooling cycle for repetitive pulse laser heating using Laplace transformation method. It is found that the magnitude of the maximum surface temperature is influenced by the cooling period of two successive pulses. The rapid response of material to heating pulses is more pronounced in the region just below the surface. Nath et al. [5,6] presented analytical solutions for the temperature profiles of heating and cooling cycles in repetitive pulse laser irradiation, and effects of various process parameters such as laser power, beam diameter, scan speed, pulse duration, repetitive frequency and duty cycle on the laser surface hardening were studied. Their calculated results and measured temperature profiles of the repetitive pulse laser heating were in good agreement in the specimen of dimensions larger than the thermal diffusion length.

However, above studies are focusing on the one-dimensional modeling, it has a certain difference with the actual problems. In fact, there are temperature difference not only in the axial direction, but also in the radial direction, when the laser irradiates the surface of material. Nevertheless, to the best of our knowledge, a detailed analytical study of the two-dimensional or three-dimensional temperature rise induced by a repetitive pulse laser is not in the literature. For this reason, the present paper attempts to find the analytical solutions of two-dimensional modeling of

* Corresponding author.
E-mail address: custcb@126.com (J. Bi).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.08.079
0017-9310/

Nomenclature

$I(r)$	spatial distribution function of laser intensity (W/m^2)	c	specific heat (J/kg·K)
$g(t)$	temporal distribution function of the laser intensity	K	thermal conductivity (W/m·K)
$P(t)$	pulse function	T_0	initial temperature (K)
I_0	incident laser power density in the center (W/m^2)	α	absorption coefficient (cm^{-1})
r_0	radius of the laser spot (m)	A	optical absorptivity
t	time (s)	R	radius of plate (m)
$\triangle t$	duration time for heating (s)	H	thickness of bulk (m)
T	temperature (K)	$\delta(t)$	Dirac function
G	Green function of temperature (K)	J_0, J_1	Bessel functions
ρ	density (kg/m^3)		

repetitive pulse laser heating material and investigates the effects of parameters on the temperature distribution. Our results will provide a quantative relationship between them and can be used to reveal the mechanism of repetitive pulse laser heating or optimize parameters in the laser process or laser damage.

On the other hand, it should be noted that the reported paper of the pulse laser interaction with material, mostly concentrated in the pulse width which is between short pulse or ultra-short pulse laser of 10^{-8}–10^{-13} s, in this case, the high power density of the laser beam will cause the material surface to produce an instant melting and vaporization. Meanwhile, high density plasma is produced and it will absorb subsequent laser energy, then the laser energy utilization efficiency is greatly reduced. To avoid the above-mentioned problems, the pulse width of millisecond magnitude laser has been risen recently [7–12], the output millisecond laser beam not only can reduce the losses in the transmission process, but also can avoid the plasma and its associated phenomena such as shock waves in the interaction process, so it improves the coupling efficiency between the laser energy and materials effectively.

This paper is devoted to two-dimensional modeling of repetitive long pulse laser heating material by analytical solution. It is organized as follows. In Section 2 we introduce the mathematical modeling and analytical solution of the temperature distributions induced by a repetitive long pulse laser heating. Results of temperature distribution for different parameters are presented in Section 3. Our main conclusions are summarized in Section 4.

2. Mathematical modeling

2.1. Repetitive laser

It is assumed that the intensity of laser beam on the surface of the material can be described as:

$$I(r,t) = I(r)g(t) \tag{1}$$

where $I(r)$ and $g(t)$ are spatial and time distribution function of laser intensity respectively. If we assume that the spatial distribution is flat-topped, then

$$I(r) = \begin{cases} I_0 = \text{constant} & \text{if } r \leqslant r_0 \\ 0 & \text{if } r > r_0 \end{cases} \tag{2}$$

where r_0 is the laser spot radius on the surface of the material, and $g(t)$ can be written as

$$g(t) = L_1P_1(t) + L_2P_2(t) + \cdots + L_{N_P}P_{N_P}(t) \tag{3}$$

where $L_1, L_2, \cdots, L_{N_P}$ are the amplitudes of pulse, and N_P is the total pulse number. The pulse function $P_1, P_2, \cdots, P_{N_P}$ can be given by

$$\begin{aligned} P_1 &= u(t-t_1) - u[t-(t_1+\Delta t_1)] \\ P_2 &= u(t-t_2) - u[t-(t_2+\Delta t_2)] \\ &\vdots \\ P_{N_P} &= u(t-t_{N_P}) - u[t-(t_{N_P}+\Delta t_{N_P})] \end{aligned} \tag{4}$$

where $u(*)$ is the unit step function, t_j $(j = 1, 2, \cdots, N_P)$ is the time for beginning of the j-th pulse, and Δt_j $(j = 1, 2, \cdots, N_P)$ is the duration time for heating of the j-th pulse. The schematic diagram of the repetitive pulse is given in Fig. 1.

2.2. Laser heating

When the laser irradiates the surface of material, laser energy was absorbed to lead to rise of surface temperature, and the internal temperature of the material is increased by heat conduction with the surface of material. Assumed laser irradiates at the center of the material surface as shown in Fig. 2, we can establish a two-dimensional axisymmetric physical problem. It only consider that in the case of conduction limited heating process and the material remains in solid phase, the governing equation of temperature can be written as

$$\rho c \frac{\partial T(r,z,t)}{\partial t} = k\left[\frac{\partial^2 T(r,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,z,t)}{\partial r} + \frac{\partial^2 T(r,z,t)}{\partial z^2}\right] \tag{5}$$

$$T(r,z,t)|_{t=0} = T_0 \tag{6}$$

$$-k\frac{\partial T(r,z,t)}{\partial z}\bigg|_{z=0} = AI(r,t) \tag{7}$$

where A is the absorption coefficient of the surface to the laser, ρ is the material density, c is the heat capacity and k is the thermal conductivity, we assume that the material parameters are not changed with temperature. It is assumed that the boundary condition is adiabatic as

$$-k\frac{\partial T(r,z,t)}{\partial r}\bigg|_{r=R} = -k\frac{\partial T(r,z,t)}{\partial z}\bigg|_{z=h} = 0 \tag{8}$$

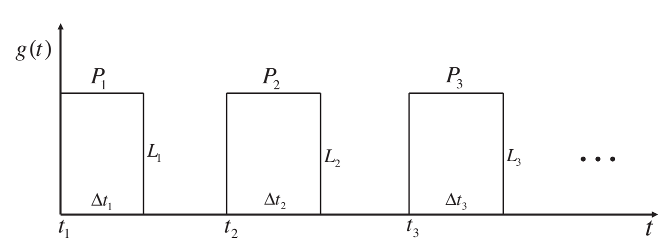

Fig. 1. Schematic diagram of the repetitive pulse.