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A multiple-relaxation-time lattice Boltzmann model for the flow and heat transfer in a hydrodynamically and thermally anisotropic porous medium

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ABSTRACT

A multiple-relaxation-time (MRT) lattice Boltzmann (LB) model is proposed for simulating flow and heat transfer in the hydrodynamically and thermally anisotropic porous medium at the representative-elementary-volume (REV) scale. By selecting the appropriate equilibrium distributions, relaxation matrix and discrete force/heat source terms, the present MRT LB model can recover the correct Darcy–Brinkman–Forchheimer and energy equations with anisotropic permeability and thermal conductivity through the Chapman–Enskog procedure. Several natural convection problems in anisotropic porous medium are simulated to validate the present LB model. The corresponding numerical results are in good agreement with data in the available literature. Especially, natural convection in a cavity with two anisotropic porous layers is investigated. The numerical results indicates that, the use of anisotropic porous layer with some optimal parameters can produce higher rate of heat transfer compared with the isotropic porous layer.

1. Introduction

Flow through the porous media with heat transfer can be found in many engineering and science fields, ranging from electronic devices cooling, drying cooling, thermal insulation, to solar collectors, nuclear reactors, food engineering and chemical engineering. People had done many attempts to construct some theoretical models to describe these porous media flows [1]. For the pore scale-models, more detailed information on flow field can be obtained. At the REV scale, the Darcy, the Brinkman-extended Darcy, and the Forchheimer-extended Darcy models are usually used. Because it is difficult to obtain the exact solutions of these governing equations, numerical simulation technique has become an important tool to solve the porous media flow problems.

In recent year, LBM has received many attentions from CFD community. Via simple collision and streaming steps, LBM can be used to simulate many complex fluid systems. Now LBM has been successfully applied to multiphase flows [2,3], turbulence flows [4,5], micro flows [6,7], fluid-interactions [8–10] and thermal flows [14,11–13]. It should be noted that LBM has been also extended to study the porous media flow. In fact, like the conventional methods, two types of LB models have been constructed at different scales. Thanks to the bounce-back rule, LBM is very suitable to simulate the flow at the pore scale. As early as 1988, Succi et al. had applied LBM to simulate three-dimensional flows in complex geometries [15]. In their work, the Darcy's law for the low Reynolds number flows was validated. Moreover, the pore-scale LB models had been used to modify the Kozeny–Carman law and Ergun's equation [16,17]. These studies indicated that pore-scale LB simulation is an excellent method to explore new physical law governing porous flows. However, when pore-scale LB models are used to simulate the practical porous media flow, the huge computational overhead may be needed. As a result, several LB models at REV scale have been proposed. Spaid and Phelan proposed a LB model with modified equilibrium distribution function for Brinkmann equation. Using their model, the steady transverse flow through a square array of porous cylinders of elliptical cross section was simulated successfully [18]. Kang et al. developed a unified LB model for flow in porous media where multiple length scales coexist [19]. Later Guo and Zhao presented a generalized LB model which considers the nonlinear inertia effect [20]. Through including the porosity into the equilibrium distribution, and adding a force term to the LB evolution equation to account

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for the linear and nonlinear drag forces of the medium, the generalized Navier–Stokes equation can be recovered. This LB model has a great influence on many subsequent studies. Chen et al. also developed a generalized LB model for flow through tight porous media with Klinkenberg's effect [21]. Hu et al. proposed a finite-volume method with LB flux solver for the Darcy–Brinkman–Forchhermer equation. In their method, the non-uniform mesh can be used to capture the thin velocity boundary layer [22]. Furthermore, some thermal LB models for heat transfer in porous media are also proposed. Guo and Zhao considered a double-distribution-function LB scheme to simulate the convective heat transfer [23]. Seta et al. did a similar work and simulated the natural convection in porous media [24]. Gao et al. developed a thermal LB model for natural convection in porous media under local thermal non-equilibrium conditions [25]. Two new distribution functions are introduced to simulate the temperature fields of the fluid and solid matrix phases. Wang et al. presented a modified LBGK model for convection heat transfer in porous media. By choosing a modified equilibrium distribution function and a source term, the macroscopic equations can be correctly recovered [26]. Numerical tests indicated this LBGK model has better numerical stability. It should be noted that the above LB models are all constructed based on the single-relaxation-time model. Liu et al. developed a multiple-relaxation-time (MRT) LB model for simulating convection heat transfer in porous media. The key point of the MRT model is to introduce the porosity into the equilibrium moments and force terms in the moment space [27]. In fact, as pointed out by Luo et al. [28], the MRT model is superior to the LBGK model in terms of accuracy, stability, and computational efficiency.

To the best of our knowledge, the existing LB models (SRT or MRT) are constructed for flow and heat transfer in only the homogeneous isotropic porous medium. Note that the study of the flow with heat transfer in anisotropic porous medium is very important due to its wide range of applications, such as drying of food grains, flow in mushy region of a solidifying alloy and flow past rod bundles in nuclear reactor core. In such case, the flow permeability and thermal conductivity must be represented using two tensors. Some works on convection in anisotropic porous medium have been done. Bruschke and Advani studied the mold filling process in anisotropic porous media using the finite element/control volume method [29]. Ni and Beckermann analyzed natural convection in a vertical enclosure filled with anisotropic porous media [30]. Degan et al. presented an analytical and numerical study of natural convection in a fluid saturated anisotropic porous medium filled in a rectangular cavity [31]. Their work studied the effects of the Rayleigh–Darcy number, anisotropic thermal conductivity ratio, anisotropic permeability ratio and the inclination angle of principal axes of the anisotropy in the permeability on the flow structure and heat transfer. Degan and Vasseur also investigated the natural convection heat transfer in a vertical anisotropic porous layer analytically and numerically [32]. They derived an analytical solution which is valid in the boundary-layer regime. In the above studies, the principal directions of the permeability are assumed to coincide with the coordinate axes. Nithiarasu et al. studied firstly the generalized cases [33]. They employed a semi-implicit, Galerkin, velocity correction procedure to solve the governing equations. The numerical results for different inclinations of the principal permeability directions were given. Krishna simulated the natural convection in a heat generating hydrodynamically and thermally anisotropic porous medium [34]. Especially, they investigated the effect of ratio of Forchheimer constants. Harfash and Hill developed a model for three dimensional double-diffusive convection in an anisotropic porous layer with a constant through flow and explored the flow instability [35].

In this paper, we aim to develop a multiple-relaxation-time model for the flows and heat transfer in a hydrodynamically and thermally anisotropic porous medium at the REV scale. By selecting the appropriate equilibrium distributions and discrete force/heat source terms, the present MRT lattice Boltzmann model can recover the governing equations with anisotropic permeability and thermal conductivity with no deviation terms through the Chapman–Enskog procedure. To validate the present model, several numerical tests are simulated and the results are compared with the date in the previous literatures.

2. Governing equations and the corresponding lattice Boltzmann model

2.1. Governing equations for the flow and heat transfer in the anisotropic porous media

In this paper, the porous medium is assumed to be hydrodynamically and thermally anisotropic. It should be noted that only the two-dimensional case will be considered. The extension to the three-dimensional situation is straightforward. The set of the macroscopic equations based on volume-averaged-theory for the flow and heat transfer under local thermal equilibrium conditions can be written as [33,34]:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\left(\frac{\mathbf{u}}{\epsilon}\right) = -\frac{1}{\rho_f}\nabla(\epsilon p) + \nu_e \nabla^2 \mathbf{u} + \mathbf{f}, \tag{2}$$

$$\rho_f c_{p,f}\left(\frac{\partial(\chi T)}{\partial t} + \nabla\cdot(\mathbf{u}T)\right) = \nabla\cdot(\boldsymbol{\kappa}_e \nabla T) + q, \tag{3}$$

where $\mathbf{u}, p$ and T are the volume-averaged velocity, pressure and temperature of the fluid, respectively. ϵ is the porosity of the porous medium. ν_e denotes the effective viscosity which is not necessarily equal to the fluid viscosity ν. ρ_f and $c_{p,f}$ are the density and specific heat of the fluid, respectively. χ is equal to $\epsilon + (1-\epsilon)(\rho_s c_{p,s})/(\rho_f c_{p,f})$. Here ρ_s and $\rho c_{p,s}$ are the density and specific heat of the solid phase. $\boldsymbol{\kappa}_e$ is the effective thermal conductivity tensor. q is the heat source. $\mathbf{f}$ is the total body force due to the porous medium and other external force fields, and is given as

$$\mathbf{f} = (f_x, f_y)^T = -\epsilon\nu\mathbf{K}^{-1}\mathbf{u} - \epsilon\mathbf{B}|\mathbf{u}|\mathbf{u} + \epsilon\mathbf{G}, \tag{4}$$

where $\mathbf{K}$ and $\mathbf{B}$ are the permeability and Forchheimer coefficient tensor. Note that when $\mathbf{K}, \mathbf{B}$ and $\boldsymbol{\kappa}_e$ are the multiples of the identity

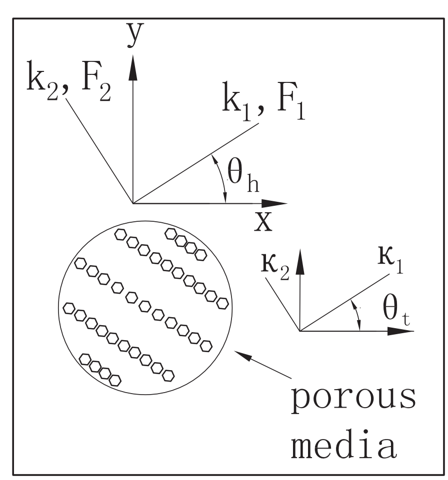

Fig. 1. Schematic of a cavity filled with the anisotropic porous medium.